doi: 10.36689/uhk/hed/2025-01-020 HED Hradec Economic Days

Batch Processing Problem – Case Study

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Abstract: The article deals with the problem of charging electric vehicles, specifically scheduling of these vehicles at charging stations. The problem is qualified as a batch processing problem on several processors, where the processor is a charging station and the batch is an electric vehicle with a charging time requirement. Two types of mathematical models are proposed, the first maximizes the number of vehicles that could be included in the charging schedule, and the second model requires charging of all vehicles, with the possibility that for some, if necessary, the charging time will be reduced. The models are verified on an illustrative example.

Keywords: batch processing; batch scheduling; integer programming

JEL Classification: C44

1. Introduction

Batch processing problem is a task containing a set of *n* batches to be processed on *K* machine processors. Here it will be independent batches without the possibility of interrupting the processing with a given processing time, with the fact that they can be processed on any machine. Machines are independent simultaneous processors. The goal is to assign batches to machines and schedule them.

The batch processing problem exists in various modifications, an overview of them can be found in (Blazewicz et al., 1997) and (Leung, 2004), where there is also the form of various limiting conditions for batch processing and the classification of these tasks is given here. An example case study can be found in (Pelikán, 2011).

2. Electric Vehicle Charging Problem

The subject of the article is the problem of scheduling electric vehicles at charging stations. Charging takes place at charging stations and this process can take from tens of minutes to hours depending on the type of charging stations and the degree of discharge of the cars. We will solve this problem at non-public stations, such as parking garages or corporate parking place, as well as closed group of vehicles.

Let us have n vehicles that require charging in a certain time interval when the driver is not using the vehicle (vehicles are in a business or parks in a reserved parking place). Its charging schedule is determined by the required charging time. The number of charging stations is given, they may also differ in charging speed. It is necessary to assign vehicles to charging stations to determine the start time of charging.

The result of the optimization is the assignment of vehicles to charging stations and the start time of charging. The following models will also solve the case where vehicle time

windows, charging requirements, and charging station capacities are such that there is no solution to this task. In that case, two solutions are proposed, the first admits the possibility that not all vehicles need to be charged (so some will be excluded from charging) and the second solution, in which all vehicles will be charged, but for some the time requirement for charging will be shortened, so they will be charged but for a shorter time than required (i.e. with a reduction of the required charging time).

3. Model with Maximum Number of Charging Vehicles (Model A)

The model does not assume that the vehicles requesting charging with their requirements for charging time and the time window for charging will be such that all vehicles can be charged for a given capacity of charging stations. If the capacity of the stations is sufficient, the model will propose a charging time schedule that includes all vehicles, otherwise it will include only a part of them and the largest number in the charging schedule.

We will use the following information about the vehicles and charging stations (parameters of the model A):

- *n* the number of vehicles;
- *K* the number of charging stations;
- d_{ik} the required charging time of the vehicle *i* if it is charged at station *k*. It is derived from energy requirement (kWh) and charging speed of station, the actual charging time depends on which station the vehicle will be charged at. Each vehicle is only charged at one of the *k* stations.
- $< t_i^0, t_i^1 >$ the time window of the vehicle *i* in which charging must be carried out;
- *M* is a large number.

Variables of the model:

- x_{ij} is binary variable, it is one if vehicle *i* is charged before vehicle *j* (not necessary immediately). It should be noted, however, this variable has different meaning than the same variable in traveling salesmen problem;
- y_{ik} is binary, value 1 means that vehicle *i* is charged at station *k*;
- $t_i \geq 0$ is time in which the charging of the vehicle *i* is started;
- z_i is a binary variable, it is one if vehicle *i* is charged at some station, otherwise it is zero.

Model A:

$$\sum_{i=1}^{n} z_i \quad \rightarrow \quad max \tag{1}$$

$$x_{ij} + x_{ji} = 1, \quad \forall\ i < j, \tag{2}$$

$$u_i + 1 - M\ (1 - x_{ij}) \leq u_j, \quad \forall\ i \neq j, \tag{3}$$

$$\sum_{k=1}^{K} y_{ik} = z_i \ , \qquad \forall\, i \tag{4}$$

$$t_i + d_{ik} - M\left(3 - x_{ij} - y_{ik} - y_{jk}\right) \leq t_j, \ \forall\, i \neq j, \ \forall\ k \tag{5}$$

$$t_i^0 \leq t_i + \sum_{k=1}^{K} d_{ik}\, y_{ik} \leq t_i^1, \ \forall\ i \tag{6}$$

The objective function (1) represents the number of charged vehicles, it will be maximized. Condition (2) ensures that either vehicle *i* is charged before the vehicle *j* (x_{ij} = 1) or the vehicle *j* is charger before the vehicle *i* $(x_{ji} = 1)$. However, condition (2) needs to be supplemented by condition (3), which prevents cyclic order of vehicles. It is an anti-cyclical condition similar to the traveling salesman problem. Conditions (2) and (3) generate the order of vehicles in which they are charged.

The time condition (5) ensures that the start time of charging of the vehicle *j* must be greater than the start time of charging the vehicle *i* plus the charging time, i.e. the start time of the charging of the vehicle *j* must follow the completion of the charging of the previous vehicle *i*. But this only applies if the *i* vehicle precedes the *j* vehicle $(x_{ij} = 1)$ and the vehicle *i* is assigned to the same station as the vehicle *j* $(y_{ik} = y_{jk} = 1)$. Thus, if all three binary variables y_{ik}, y_{jk} and x_{ij} are one, their sum is 3. If this sum is less than 3, then all these conditions are not met and a large number M is subtracted from the left-hand side of (5) thanks to therefore, condition (5) does not affect on the value t_j on the right-hand side of the inequality (5).

Condition (6) is the condition of the time window of vehicle *i*, which ensures that the charging of the vehicle *i* takes place within the time window $< t_i^0, t_i^1 >$.

Condition (4) determines the value of the binary variable z_i which is equal to one if the vehicle *i* is charged at any of the charging stations, otherwise it is equal to zero. At the same time, it prevents the vehicle from being charged on multiple stations.

4. Model with the Condition of Charging All Vehicles (Model B and C)

All vehicles will be charged in these models. If the capacities of the stations would not allow it, the model allows for a reduction in charging requirements. The reduction in the charging time d_{ik} of vehicle *i* at station *k* will be reduced to $r_i d_{ik}$, where the variable $r_i \in (0,1 >$ is a variable of the model. If the reduction r_i is one, the requirement d_{ik} will not be reduced, otherwise these reductions will have to be brought as close as possible to the value one. In model B, we will maximize *Rmin*, the variable with the smallest value of these reductions r_i. We then use this optimum value in model C, where the problem is to find the r_i in such a way that the requirements of the vehicles are achieved as much as possible and at the same time the capacity of the charging stations is used to the maximum.

The solution proceeds in two phases: in the first phase (model B) we obtain the maximum value of *Rmin* for which it is possible to ensure the charging of all vehicles, in the second phase we maximize the reductions (as the sum of r_i) in order to achieve higher values of r_i.

Model B contains the following additional variables compared to model A:

- $r_i \in (0,1>$ reduction in vehicle demand *i*;
- *Rmin* ≥ 0 maximum minimum reductions r_i;
- $dr_i \geq 0$ reduced vehicle charging time of the vehicle *i*.

The constraints define the order of vehicles are the same in model A (2)-(3) and in the model B (8), (9) and in the model C (16), (17). Unlike the model A equation (10) in model B and (18) in model C requires charging all vehicles. Furthermore, conditions (11), (12) are a modification of conditions (5), (6) in the model A where instead of the charging requirement d_{ik}, a reduced requirement dr_i is inserted in these inequalities.

Inequality (13) of the model B defines the reduced charging requirement dr_i, is the same in the model C (21).

Model C contains the parameter *Rmin*, which is the optimal value of the object function of the model B, this parameter ensures the minimum of all reductions r_i in the inequality (22). The sum of the reductions r_i is the objective function of the model C.

Model B:

$$Rmin \rightarrow max \tag{7}$$

$$x_{ij} + x_{ji} = 1 \quad \forall\ i < j, \tag{8}$$

$$u_i + 1 - M\,(1 - x_{ij}) \leq u_j,\ \forall\ i \neq j, \tag{9}$$

$$\sum_{k=1}^{K} y_{ik} = 1, \qquad \forall\ i \tag{10}$$

$$t_i + dr_i - M\,(3 - x_{ij} - y_{ik} - y_{jk}) \leq t_j, \qquad \forall\ i \neq j,\ \forall\ k \tag{11}$$

$$t_i^0 \leq t_i + dr_i \leq t_i^1,\ \forall\ i \tag{12}$$

$$d_{ik} r_i - M(1 - y_{ik}) \leq dr_i \leq d_{ik} r_i + M(1 - y_{ik}),\ \forall i, \forall k \tag{13}$$

$$Rmin \leq r_i \leq 1, \qquad \forall i \tag{14}$$

Model C:

$$\sum_{i=1}^{n} r_i \rightarrow max \tag{15}$$

$$x_{ij} + x_{ji} = 1, \quad \forall\ i < j, \tag{16}$$

$$u_i + 1 - M\,(1 - x_{ij}) \leq u_j, \quad \forall\ i \neq j, \tag{17}$$

$$\sum_{k=1}^{K} y_{ik} = 1, \quad \forall\ i \tag{18}$$

$$t_i + dr_i - M\left(3 - x_{ij} - y_{ik} - y_{jk}\right) \leq t_j,\ \forall\ i \neq j,\ \forall\ k \tag{19}$$

$$t_i^0 \leq t_i + dr_i \leq t_i^1,\ \forall\ i \tag{20}$$

$$d_{ik}r_i - M(1 - y_{ik}) \leq dr_i \leq d_{ik}r_i + M(1 - y_{ik}),\ \forall i, \forall k \tag{21}$$

$$Rmin \leq r_i \leq 1, \quad \forall i \tag{22}$$

5. Example

Let's have 5 vehicles and 2 charging stations. The vehicle requirements for charging and time windows for charging are listed in Table 1.

Table 1. Example – data

i	1	2	3	4	5
d_{i1}	2	1	9	8	4
d_{i2}	2	6	3	8	4
$t_i^0 - t_i^1$	8-12	8-14	12-20	13-20	15-23

Table 2 shows the results of the optimal solution of the model A. In this solution, vehicle 4 is not charged for capacity reasons, for other vehicles that are being charged, the start time of charging is indicated in line t_i, and the end of charging in the next line.

Table 2. Optimal solution: result of model A

i	1	2	3	4	5
k	1	1	2	-	1
t_i	8	10	12	-	15
$t_i + d_{ik}$	10	11	15	-	19

Table 3. Optimal solution: result of model B

i	1	2	3	4	5
k	1	1	2	1	2
t_i	8.875	8	12	13	15
r_i	0.875	0.875	0.875	0.875	0.875

Table 4. Optimal solution: result of model C

i	1	2	3	4	5
k	1	1	2	1	2
t_i	8	10	12	13	15
r_i	1	1	1	0.875	1

Optimal solution of model B involves charging all vehicles, but charging times have been reduced for all of them. The optimal value *Rmin* = 0.875. The values of variables r_i are not maximized in the model B, so we will use the model C to maximize them. In Table 3, all r_i

values are equal to 0.875. If we use this value as a lower limit for r_i in the model C, the values r_i for vehicles 1, 2, 3, 5 have increased to the value 1 (see Table 4).

6. Conclusions

The presented models represent a proposal for modeling the problem of scheduling electric vehicles during charging. These models can easily be supplemented with other charging conditions, e.g. vehicle priorities, vehicle handling times, etc.

Conflict of interest: none

References

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