Analysis of the Manufacturing Industry Using the Cobb-Douglas Production Function

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Abstract: The emergence of Industry 4.0 presents new challenges and opportunities in the field of economics and labor productivity. A key aspect of this transformation lies in the ability to accurately analyze the substitution of labor by capital, which is necessary for sound decision-making in managerial practice and for the formulation of policies that support innovation and economic growth. The Cobb-Douglas production method provides a robust analytical tool for examining this relationship within the context of Industry 4.0 and its impact on strategic decision-making regarding human resources and technology investments. The article aims to create a Cobb-Douglas production function for the manufacturing industry, both for the entire industry and for individual sectors in the Czech Republic. First, a correlation analysis will be performed, then the Cobb-Douglas production function will be constructed, and the relevant coefficients will be calculated using the least squares method. The degree of determination will then be verified for the entire model. The process will be carried out for individual manufacturing industry sectors in the next part. The different degrees of determination will be discussed in individual sectors and the whole.

Keywords: industry; correlation analysis; Cobb-Douglas production function; method of least squares; Industry 4.0

JEL Classification: D81; C69

1. Introduction

Industry plays an irreplaceable role in the Czech economy, accounting for almost a third of the total gross added value, which is the highest share in the EU28 countries. Despite the growing share of services in developed countries, the importance of the manufacturing industry in the Czech Republic remains significant. The manufacturing industry of the Czech Republic is highly developed and its sector accounts for roughly 23% of economic output (share in the creation of gross added value in 2022). The Czech Republic leads among European countries in the share of the manufacturing industry in gross added value, even surpassing Germany, Slovakia and Poland. Almost a quarter of total employment in the Czech Republic is in the manufacturing industry, which is also among the highest values in Europe (Ministerstvo průmyslu a obchodu, 2023).

The manufacturing industry in the Czech Republic is significantly export-oriented, which emphasizes its key role in the country's economy. The growth of foreign demand is very important for the Czech economy.

The Cobb Douglas production function is one of the key concepts of economic theory that is used to model the production process in industries. This function is particularly relevant in the context of the manufacturing industry, which represents a significant part of the economy of the Czech Republic.

Industry 4.0, as a modern initiative aimed at the automation and digitization of industrial processes, has a significant impact on the way production processes are organized in the Czech manufacturing industry. This trend encourages the substitution of labor for new technologies such as robotization, automation, and artificial intelligence, which affects the parameters of production functions (Hedvičáková & Král, 2021; Maresova et al., 2018).

The elasticity of substitution between labor and capital, referred to as σ (sigma), is used by economic theory as a measure of the substitutability of these factors in the production process. Early studies suggested that σ could be around 1, but more recent research presents mixed results. Estimates of the elasticity of substitution are sensitive to the methodology used and the data available (Procházková Ilinitchi et al., 2021).

The degree to replace capital and labor factor connections offers many variables (Chirinko, 2002; Knoblach & Stöckl, 2020) and there is a debate (Chirinko, 2008) about estimates of σ based on different short-run and long-run models, returns to productive factors in an open economy (Jones & Ruffin , 2008; Knoblach & Stöckl, 2020), the relationship between technology shocks and hours worked (Cantore et al., 2017; Knoblach & Stöckl, 2020), as well as industry transformation (Alvarez-Cuadrado et al., 2017; Knoblach & Stöckl, 2020).

2. Methodology

2.1. Correlation Analysis

A Spearman correlation coefficient is an important characteristic in evaluating the validity of tests because it determines how closely two related phenomena are captured together. Thus, it allows quantitative determination of how far the two similar orders are created. For the calculation, it is necessary to have a table in which you can specify individual correlated pairs, which are compared to the individual components of the correlation, the overall index, and the basic form of vector analysis. The result is a dimensionless number, which indicates the degree of correlation between individual freedom and the steam created for each pair of correlations.

2.2. Cobb-Douglas Product Function

In economics, the Cobb-Douglas production function is widely used to represent the relationship between inputs and outputs. It was proposed by the Swedish economist Knut Wicksell, who lived from 1851–1926, and tested by Charles Cobb and Paul Douglas in 1928. The Cobb-Douglas production function is a production function in the long run. In 1928, Charles Cobb and Paul Douglas published a study in which they modeled the growth of the

American economy from 1899–1922. They considered a simplified view of the economy in which the amount of labor and capital invested determines production output. Although many other factors influence economic performance, their model has proven remarkably accurate. The Cobb-Douglas production function is of the form (Cobb & Douglas, 1928; Hušek, 2007):

$$Q(L, K) = AK^{\alpha}L^{\beta}, \tag{1}$$

where

Q is the total output,

L is the labor input,

K is the capital input,

A is the technology level,

α is the elasticity of production relative to labor input,

β is the elasticity of production concerning capital input,

A, α, β are positive constants.

2.3. Method of Least Squares

One of the most widely used methods of estimating the production function is the least squares method, in which the function that leads to the smallest sum of squares of the deviations of the observed values of the dependent variable from the theoretical values calculated from the derived point estimation function is considered the most appropriate. The least squares method is a mathematical-statistical method and is particularly suitable for processing data obtained by measurement. It can also be used to find the Cobb-Douglas production function from the input data. First, we need to find a linear relationship between the unknown parameters. We take the natural logarithm of both sides from formula (1) to do this.

$$ln(Q) = ln(A) + \alpha\, ln(K) + \beta\, ln(L) \tag{2}$$

For the equation to make sense, the values of K, L, and Q must be positive, which always satisfies (we cannot have a negative number of workers, machines, and production must always be equal to zero). If we introduce the substitution

$$a = ln(A)\,, x = ln(K)\,, y = ln(L)\,, z = ln(Q)$$

then the Cobb-Douglas production function can be rewritten as a linear economic metric model

$$z = a + \alpha x + \beta y, \tag{3}$$

Now, we will use the least squares method (OLS) to find suitable values of a, α, β. The essence of the least squares method is to determine the appropriate observation function y given the known observation matrix X to obtain the best estimates of the model's unknown parameters. If we limit ourselves to linear transformations y, then for a point linear estimation function or statistic, we can write b = Ay, where b is the column vector of the estimation of a, α, β, and A is a k × x matrix. More in (Šubrt, 2011).

After de-logarithming and substituting into the Cobb-Douglas production function, we get its resulting mathematical form for the given subject.

2.3. Coefficient of Determination

The coefficient of determination, commonly referred to as R^2 is a measure of the quality of a regression model in mathematical statistics, which in its basic form expresses what proportion of the variability of the dependent variable the model explains. The coefficient of determination can take on a maximum value of 1 (or expressed as a percentage of 100%), which means a perfect prediction of the values of the dependent variable. Conversely, a value of 0 (or 0%) means that the model does not provide any information for the knowledge of the dependent variable; it is completely useless. The coefficient of determination of a linear regression model is usually defined as one minus the quotient of the error variance (i.e., the differences between the model's predictions and the true values of the independent variable) and the variance of the independent variable (Salh, 2015; Yin, 2001).

3. Analysis and Results

3.1. Data Description

We obtained data for the manufacturing industry from the Ministry of Industry and Trade website (Ministerstvo průmyslu a obchodu České republiky, 2024); we obtained the main macroeconomic indicators from the website of the Czech Statistical Office (Český statistický úřad, 2024).

For the purposes of the Cobb-Douglas production function and Least Squares processing, we selected the following economic indicators: labour costs, investments, and EBIT. Due to data availability, we examined the development in the years 2008-2021; see Table 1.

Table 1. Labour costs, investments, and EBIT in 2008-2021

Year	Labour costs	Investments	EBIT
2008	315,710,942,000	225,851,380,000	188,386,692,000
2009	283,926,078,000	148,303,544,000	132,525,605,000
2010	286,905,981,000	142,894,670,000	204,227,236,000
2011	303,852,425,000	161,348,607,000	215,494,865,000
2012	313,559,542,000	177,572,810,000	216,890,602,000
2013	314,012,504,000	183,179,062,000	226,601,621,000
2014	328,840,762,000	208,429,860,000	302,690,252,000
2015	349,111,463,000	220,053,605,000	329,430,302,000
2016	373,513,341,000	215,652,485,000	314,898,375,000
2017	407,734,253,000	242,651,396,000	328,429,987,000
2018	443,078,891,000	281,135,918,000	311,888,110,000
2019	465,277,486,000	267,838,873,000	305,919,835,000
2020	456,103,533,000	233,466,350,000	239,114,992,000
2021	478,943,460,000	243,643,490,000	347,579,776,000

In the following Table 2, basic statistical characteristics are calculated for these selected indicators. From Table 2 can be seen that the smallest standard deviation can be observed for investments.

Table 2. Basic statistical characteristics for the investigated variables

Year	Labour costs	Investments	EBIT
count	14	14	14
mean	365,755,047,214.28	210,858,717,857.14	261,719,875,000.00
std	70,695,142,142.82	42,947,353,035.16	66,012,168,414.20
min	283,926,078,000.00	142,894,670,000.00	132,525,605,000.00
25%	313,672,782,500.00	178,974,373,000.00	215,843,799,250.00
50%	338,976,112,500.00	217,853,045,000.00	270,902,622,000.00
75%	434,242,731,500.00	240,355,134,500.00	314,145,808,750.00
max	478,943,460,000.00	281,135,918,000.00	347,579,776,000.00

The development is illustrated in the following graph (Figure 1). From the graph, you can see the decrease of the investigated quantities in 2020 due to the Covid-19 pandemic. The biggest drop in 2020 can be seen in EBIT.

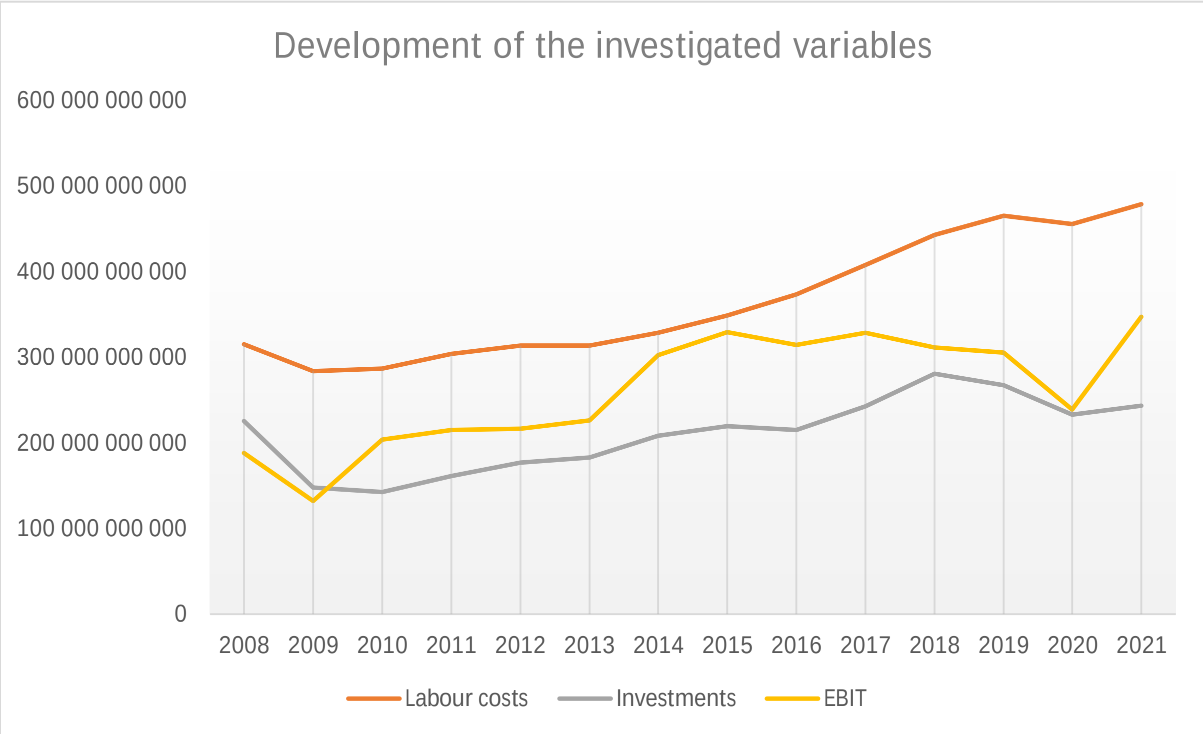

Figure 1. Development of investigated variables in 2008-2021

3.2. Correlation Analysis

From the following results of the correlation analysis, it can be seen that the investigated quantities are highly interconnected, which is not harmful for use in the Cobb-Douglas production function, since a relationship between the given quantities is assumed (see Table 3).

Table 3. Correlation analysis for the investigated variables

	Labour costs	Investments	EBIT
Labour costs	1.00000	0.86467	0.69759
Investments	0.86467	1.00000	0.72959
EBIT	0.69759	0.72959	1.00000

The degree of correlation is illustrated in the following graph (see Figure 2).

Figure 2. Correlation analysis for the investigated variables

3.3. Least Squares Method

Using the least squares method, applied to the Cobb-Douglas production function, we got the following elasticities (see Table 4):

Table 4. Results of elasticity coefficients computed using OLS method.

	Elasticity
α = ***elasticity for labour costs***	0.35415402846865335
β = ***elasticity for investments***	0.6461906128892037

3.4. Results for Cobb-Douglas Production Function

The following figures show the results of the Cobb-Douglas function. The first image Figure 3 below shows the surface for the natural logarithm, where the exact fit of the input points can be seen.

After de-logarithming and substituting into the Cobb-Douglas production function, we get its resulting mathematical form for the given subject. Figure 4 shows the surface after de-logarithming. Since many points are further away from the calculated surface, a smaller determination index can be expected for a given model.

3.5. Index of Determination for the Manufacturing Industry

For the created Cobb-Douglas production function model, we calculated the determination index:

$$R^2 = 0.548907753885556$$

Since this index is not very high, we wanted to find out which industries contribute the most to this result. Therefore, in the following section, we calculated the Cobb-Douglas production function for all sectors of the manufacturing industry and compared the values.

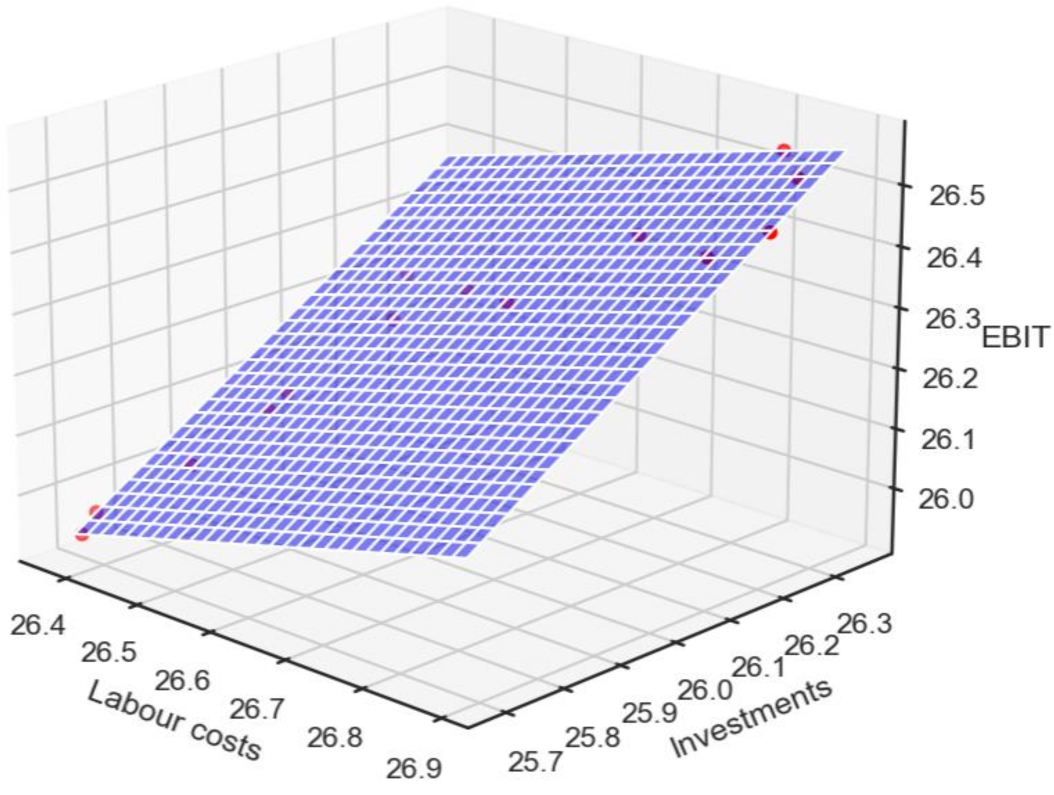

Figure 3. Results of the Cobb-Douglas function – logarithmic

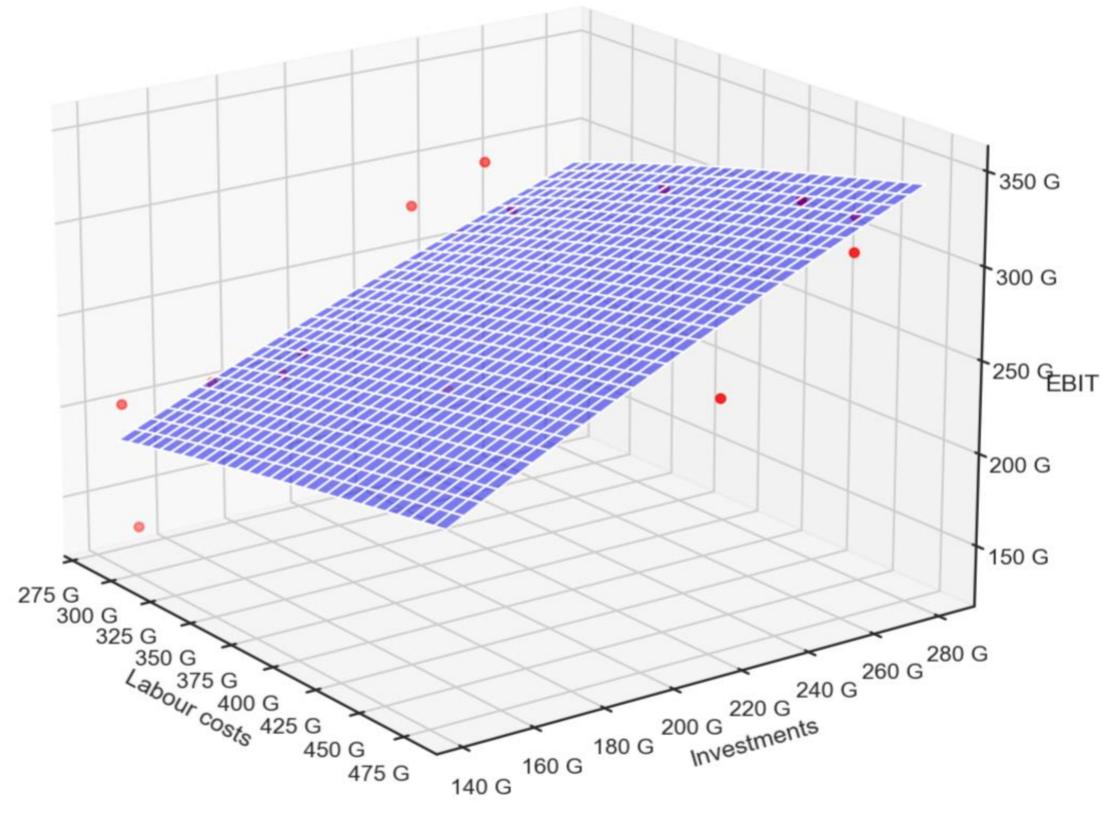

Figure 4. Results of the Cobb-Douglas function – final

3.6. Results for Individual Sectors of the Manufacturing Industry

The following Table 5 shows the results for individual sectors of the manufacturing industry, sorted by the coefficient of determination.

Table 5. Results for individual sectors of the manufacturing industry

	α	β	R^2
Manufacture of metal structures and fabricated metal products, except machinery and equipment	0.81	0.2	0.82
Other manufacturing	0.14	0.84	0.78
Production of other non-metallic mineral products	0.17	0.82	0.75
Manufacture of paper and paper products	0.09	0.9	0.55
Production of beverages	0.37	0.65	0.54
Manufacture of food products	0.33	0.64	0.52
Wood processing, manufacture of wooden, cork, wicker, and straw products, except furniture	0.69	0.32	0.43
Furniture production	-0.12	1.06	0.38
Printing and reproduction of recorded media	0.35	0.64	0.28
Manufacture of motor vehicles (except motorcycles), trailers and semi-trailers	0.68	0.31	0.25
Production of chemical substances and chemical preparations	0.49	0.52	0.21
Manufacture of basic pharmaceutical products and pharmaceutical preparations	-0.14	1.14	0.21
Manufacture of leather and related products	0.28	0.65	0.05
Production of textiles	0.22	0.74	-0.01
Production of rubber and plastic products	0.13	0.87	-0.01
Production of electrical equipment	-0.72	1.65	-0.25
Manufacture of machinery and equipment	0.62	0.37	-0.49
Production of other means of transport and equipment	-0.25	1.21	-0.85

Following sectors couldn't be computed caused by the negative EBIT value, which arose due to the inclusion of the period during the Covid-19 pandemic:

- Manufacture of clothing,
- Manufacture of basic metals, metallurgical processing of metals; foundry,
- Manufacture of computers, electronic and optical instruments and equipment.

For individual sectors, the highest value of the coefficient of determination came out for Manufacture of metal structures and fabricated metal products, except machinery and equipment, on the contrary, for many sectors this value came out very low. Which was caused by the drop in the investigated quantities around 2020.

4. Discussion

Much research points to the importance of literature that suggests doubts about the use of the Cobb-Douglas production function. The authors (Gechert et al., 2022) argue that after accounting for publication bias and model uncertainty, the true value of the elasticity of substitution decreases even more, highlighting the need for a critical approach to research methods. The accuracy of the central estimate of the elasticity of substitution is questioned, especially given possible limitations and dependence on available data. Further research is suggested that would deal with more precise methods and include other determining factors. Overall, this discussion provides context for the study's results and suggests the need for further research on production functions and elasticity of substitution in economic models. Another research (Hašková et al., 2021) that compared the sector of knowledge-intensive services with the production function of the processing sector, which represents a key source of gross domestic product in the Czech Republic in the years 1995-2018. The results of this research show that even though manufacturing is one of the industries heavily dependent on physical capital, changing the capital to worker ratio in this industry has the biggest impact on output.

Other authors (Hájková & Hurník, 2007) point out that the Cobb-Douglas production function is used to analyze performance from the point of view of supply and to measure the country's production potential. However, this functional form assumes a constant share of labor in output, which may be too restrictive for a converging country. In the period 1995-2005, the authors (Hájková & Hurník, 2007) do not observe a significant difference between the calculation of the supply side of the Czech economy using the Cobb-Douglas production function and a more general production function, although the share of labor in the Czech Republic gradually increased.

The study (Husain, 2016) found that the study found that the coefficients for K and L are 0.49 and 0.51 for the entire manufacturing sector in Bangladesh, which means that labor is more productive than capital. Also, the estimated results (Zakir Hossain & Said Al-Amri, 2010) indicate that the manufacturing industry of Oman generally indicates a case of increasing returns to scale. Of the nine industries, seven show increasing returns to scale and only the remaining two show decreasing returns to scale between 1994-2007.

Our study also confirmed a higher coefficient for K (0.65) compared to the coefficient for L (0.35), which is consistent with the study mentioned above.

5. Conclusions

The aim of the article was to create a Cobb-Douglas production function for the manufacturing industry, both for the entire industry and for individual sectors. The compiled model for the entire manufacturing industry gave higher coefficient for K (0.65) compared to the coefficient for L (0.35), but the results for individual sectors were very varied.

Our study found that the coefficients for K and L are 0.65 and 0.35 for the entire manufacturing industry, which means that labor is more productive than capital.

In conclusion, it can be stated that Industry 4.0 represents an important factor in the substitution of labor by capital. This new industrial paradigm includes significant advances in automation, digitization, and the Industrial Internet of Things, which enable more efficient use of capital investment and reduce the need for human labor in some production and service processes. In this way, Industry 4.0 can contribute to increasing the productivity and competitiveness of the economy, but at the same time it can also have an impact on the labor market and require the adaptation of the workforce to new technologies and skills. It is therefore crucial to examine the impact of Industry 4.0 on the substitution of labor by capital and to prepare strategies and policies to successfully adapt to these changes in the industrial and work environment.

Acknowledgments: This contribution has been supported by the institutional support of the University of Pardubice, Czech Republic. The research/work was supported by the internal project “SPEV – Economic Impacts under the Industry 4.0, Societies 5.0 & 6.0 Concept”, 2024, University of Hradec Králové, Faculty of Informatics and Management, Czech Republic. We thank Martin Matějíček for his help.

Conflict of interest: none.

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