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Game Theory and Agent-Based Models in Epidemiology: Exploration of Strategies with NetLogo

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Abstract: The paper deals with the fusion of game theory and computational epidemiology. It presents an agent-based simulation that demonstrates how game theory principles can be used to study the effects of individual decision making in an epidemic situation. Our model of a symmetric repeated vaccination game with imperfect information was developed using NetLogo. Players decide whether or not to vaccinate by weighing the costs of vaccination against the potential costs of disease. The decisions of neighbors and the course of epidemic determine the costs. Different diseases and scenarios can be simulated by manipulating the model with input parameters. The model allows the number of infections, the number of players following a particular strategy, and the highest payouts for players in each of the three states. Four experiments were conducted.

Keywords: agent-based model; epidemiology; game theory; model; NetLogo; simulation

JEL Classification: C53; D71

1. Introduction

Despite considerable technological and medical progress in our society, we have failed to react promptly and adjust to the COVID-19 pandemic outbreak. It has become apparent that a comprehensive strategy and the resources to manage the intricacies of pandemic scenarios, the various factors influencing their progression, and human behavior are missing in our society. Our study adds to existing models of individual decision-making concerning vaccination and other anti-pandemic interventions through the application of game-theoretic principles in agent-based models.

1.1. Mathematical Modelling of Epidemics

Batista et al. (2021) argue that epidemiological models classify individuals in a population into the following categories: Susceptible (S), Exposed (E), Infected (I), Recovered (R). The model dynamics are determined by the rate of movement of an individual between categories. The most frequent transitions among these categories are:

- SIS (Susceptible – Infected – Susceptible),
- SIR (Susceptible – Infected – Recovered),
- SIRS (Susceptible – Infected – Recovered – Susceptible),
- SEIR (Susceptible – Exposed – Infected – Recovered).

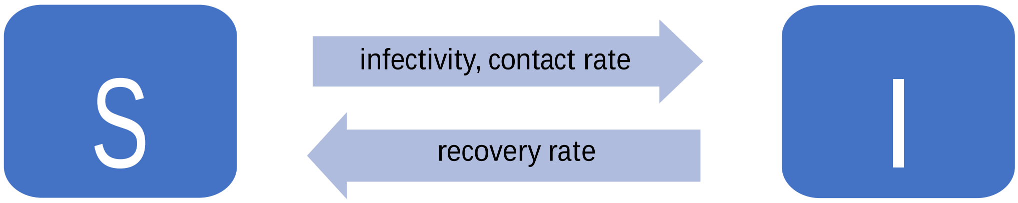

Figure 1. SIS model

The SIS model (see Figure 1) examines scenarios in which individuals have no immunity after contracting the disease, and upon recovery, they immediately return to the susceptible phase.

Mathematically, the situation can be described:

$$\frac{dS}{dt} = \Lambda - \frac{\beta SI}{S + I} - \mu S + \phi I \tag{1}$$

$$\frac{dI}{dt} = \frac{\beta SI}{S + I} - (\alpha + \mu + \phi)I \tag{2}$$

where the parameter Λ is the number of susceptible individuals, μ is the natural mortality rate of the population, β is the disease transmission coefficient, α is the disease-assisted mortality rate, and Φ is the rate of movement of an individual from one category to another and back. After adjustment, we obtain the population-wide equation written by Vargas (2011):

$$\frac{d}{dt}(S + I) = \Lambda - \mu(S + I) - \alpha I \tag{3}$$

The population size undergoes natural fluctuations over time and stabilizes at an equilibrium state in the absence of disease Λ / μ.

Vargas (2011) elucidates the SIR and SIRS epidemiological models (graphic representation in Figure 2) in which people possess either temporary or long-lasting immunity to infection and recovery. The SIR model involves susceptible and infected individuals coming into contact with each other and having a certain likelihood of becoming infected. In this instance, we observe the transition rate between the susceptible and infected states. Once healed, an individual in the infected category enters the cured category and remains there, having acquired full immunity to the illness. The SIRS model assumes only temporary immunity to the disease, which is determined by the parameter representing the duration. This model can be used, in practice, for seasonal illnesses such as influenza or COVID-19.

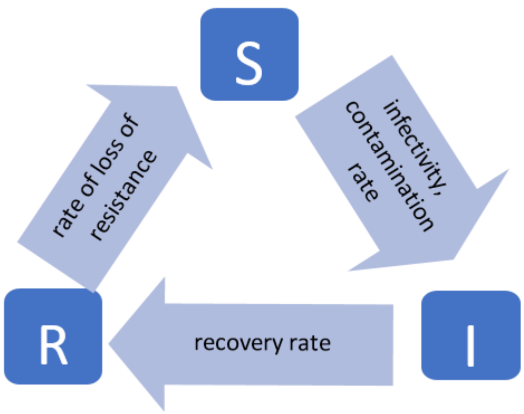

Figure 2: SIR model

The mathematical expression of each category is:

$$\frac{dS}{dt} = \Lambda - \frac{\beta SI}{S + I + R} - \mu S + \phi I \tag{4}$$

$$\frac{dI}{dt} = \frac{\beta SI}{S + I + R} - (\alpha + \mu + \phi)I \tag{5}$$

$$\frac{dR}{dt} = \kappa I - (\mu + \gamma)R \tag{6}$$

where parameters Λ, μ and β are positive constants and parameters α and γ are non-negative constants. Another assumption is that κ is a kind of constant rate of recovery of an individual and parameter γ is a constant rate of loss of immunity of an individual to the disease. The author describes time $1/\kappa$ as the mean average time of infection and time $1/\gamma$ as the mean average time of immunity. By summing these equations, a single differential equation can be obtained:

$$\frac{d}{dt}(S + I + R) = \Lambda - \mu(S + I + R) - \alpha I \tag{7}$$

Similar to equation (3), the total population size changes over time and converges to an equilibrium state in the absence of disease Λ/μ.

The SEIR model includes a category for those infected but not yet infectious, whereas the SIERS model assumes that those cured can lose immunity and become infectious gain (Camacho et al., 2020). One of the more comprehensive models for COVID-19 is SIDARTHE, comprising 8 states: Susceptible, Infected, Diagnosed, Ailing, Recognized, Threatened, Healed, Extinct (Higazy, 2020).

1.2. Game Theory in Epidemiology

Game theory is commonly used in epidemiology to investigate the tactical actions of individuals. In modeled scenarios of repeated games, individuals in a population select between strategies and adapt their strategies in subsequent rounds of the game based on their prior payoffs. Agent-based models enable the realistic simulation of diverse populations responding to various epidemic scenarios, assessing the decision-making strategies of individuals, their reliance on experience or peer influences.

Vaccination games serve as models that reflect the vaccination attitudes of individuals. Zhang et al. (2013) introduced a gaming scenario that involved three strategies: vaccination, self-protection, and laissez-faire. They found that this model can create the Braess paradox, in which a superior individual outcome results in an inferior societal outcome.

The significance of spreading information regarding the disease, vaccines, and individuals' attitudes is illustrated in a study by Kabir and Tanimoto (2019). The study integrates a two-layer model of the SIR/V-UA epidemic's spread into a metapopulation migration model for random walkers to investigate how individuals' information affects their access to vaccinations or their movement to a safer location. Players are divided into groups based on their health and vaccination status, each assigned its own payoff matrix. The susceptibility to disease varies by group, and the rate of contagion may decrease depending on available information. The authors' conclusion is that information regarding disease and its spread is critical to the epidemic management process.

Della Marca et al. (2013) employs evolutionary game theory to simulate swift decision-making dynamics and rapid shifts in opinion on the SIR model. Meanwhile, (Okita et al., 2023) present a stochastic approach utilizing evolutionary game theory to model the spread of epidemics.

The impact of incomplete information on vaccine deferment was the topic of discussion in (Bhattacharyya & Bauch, 2011). The video game was developed in response to the behavior of individuals who postponed getting the H1N1 vaccine in 2009 because of limited information concerning the vaccine's adverse effects and inadequate testing. A comparable situation has emerged in the context of the COVID-19 pandemic, where divergent viewpoints and stances on vaccination have created divisive tendencies among the populace. The prevalent utilization of social media has played a significant role in fostering social division by disseminating official and unofficial information. The SIR epidemic model categorizes individuals based on their status as vaccinated or unvaccinated, which serves as the basis for describing the model's dynamics within this framework. Each game round spans 52 weeks, during which players may choose to get vaccinated or remain unvaccinated. Vaccination offers two weeks of protection.

Getting vaccinated is important for achieving group immunity, especially for unvaccinated individuals. For those who already have group immunity and are surrounded by vaccinated individuals, information obtained about vaccine safety from their environment reduces the hypothetical vaccination cost. The common approach is to "wait and see", leading players to delay vaccination. It should be noted that in both scenarios, approximately 20% of players choose not to get vaccinated at all.

Kuga and Tanimoto (2018) present a framework for comparing two imprecise methods for safeguarding against infectious illnesses: vaccination, which provides partial protection, and mask-wearing. Technical terms are explained on their first use to assist readers in comprehension. The study utilizes the SIR (Susceptible, Infectious, Recovered) model to create a repeated game that spans multiple rounds, with each round ending after all individuals have recovered. The text maintains a formal register, precise language, and objective tone while adhering to standard style guides and conventions. The study utilizes the SIR (Susceptible, Infectious, Recovered) model to create a repeated game that spans multiple rounds, with each round ending after all individuals have recovered. After each round, an individual's strategy is influenced by the strategy of a randomly encountered individual, the average of chosen strategies in the population, or random choice. The payoff function is determined by the cost of infection (if infected) and the cost of vaccination. Two metrics were used to compare outcomes: vaccine efficacy and mask effectiveness. The results suggest that suboptimal vaccination is more effective in managing a pandemic when vaccine effectiveness and mask efficacy are equivalent.

The scientific community is investigating the feasibility of predicting or affecting the increase in vaccine deferrals. The study intends to enhance comprehension regarding the probable consequences of vaccine deferral approaches. Bhattacharyya and Bauch's (2010) research introduced a game regarding voluntary vaccination postponements against childhood illnesses, based on age and perceived risk. The game provides three choices: vaccination, deferral, or non-vaccination. As infection rates rise, those who defer vaccination and those who refuse it entirely reach a balance, as highlighted by the game's results. This tends to happen when the cost of vaccination is high or the risk of contracting the disease is low among the first age group. As a result, people tend to postpone getting vaccinated until the second age group, when vaccination costs or disease risks are lower for younger children. Earlier vaccination becomes a more favorable option as the number of infected individuals and their risk of infecting others increase. Consequently, this reduces the number of cases while increasing instances of deferral, resulting in an increase in infected individuals.

1.3. Epidemic Models in NetLogo

NetLogo (Wilensky, 1998) is a platform used to implement agent-based models in various application areas, including epidemiology:

- *EpiDEM Basic* model (Yang & Wilensky, 2011) simulates the spread of a disease in a closed population. The model is based on the Kermack-McKendrick model of SIR in a closed population over time. In the model, agents move randomly around the world, and when an infected agent encounters a susceptible agent, there is a probability that the uninfected agent will become infected.
- *Epidemic Travel and Control* model (Rand & Wilensky, 2008) simulates the spread of a disease in a semi-confined population with additional elements such as travel, isolation, quarantine, vaccination and links between individuals.
- *Spread of Diseases* model (Rand & Wilensky, 2008) deals with the spread of diseases in different settings and environments. Its main objective is to study what assumptions and interactions between agents can fundamentally affect the results of the model.
- *Virus* model (Wilensky, 1998) simulates the transmission and spread of a virus in a human population. Agents move randomly and are generated with random ages. Agents can die and reproduce.
- *Virus on Network* model (Stonedahl & Wilensky, 2008) simulates the spread of a computer network infection, with the ability to experiment with both network and virus parameters.

2. Methodology

The objective is to showcase the integration of game theory and agent-based modeling concepts in an epidemiological model using NetLogo.

The model is a multi-round, iterative, and symmetrical vaccination game with incomplete information. Players select their strategy based on a comparison of the cost of the disease and vaccination. The cost is influenced by the epidemic course as well as the decisions of adjacent players, and players can opt to vaccinate or not.

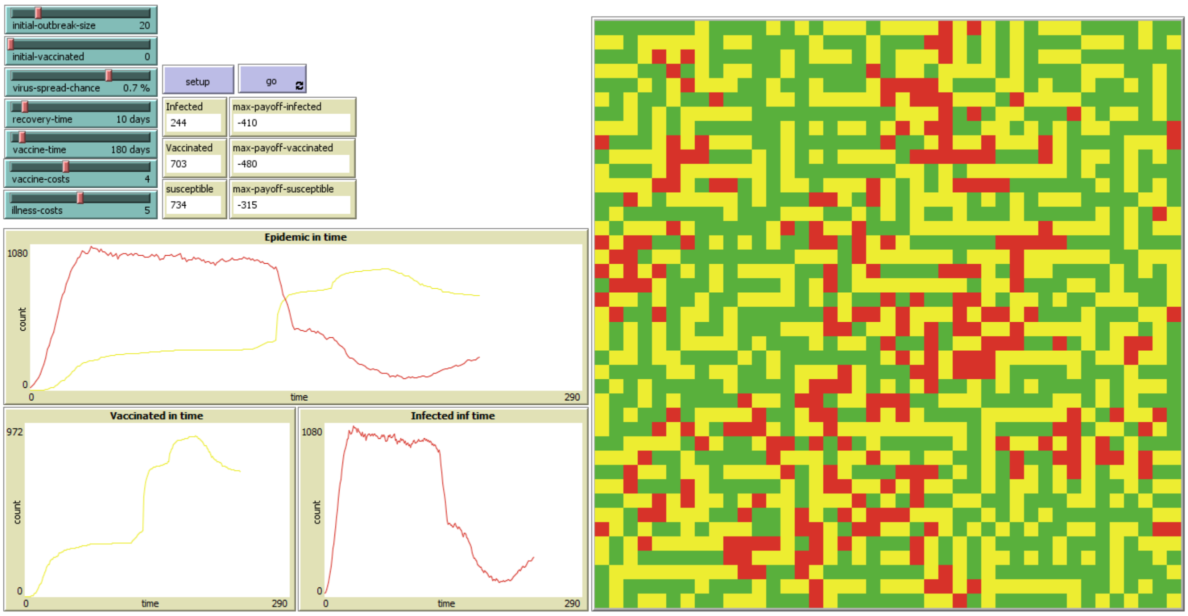

Figure 3: Netlogo user interface

The simulation is regulated by input parameters in the user interface (see Figure 3), facilitating the simulation of varied scenarios and diseases. Seven parameters are defined as follows:

- The *initial outbreak size* parameter denotes the count of persons infected at the onset of a certain epidemic. Parameter takes values between 1 to 100.
- The *initial vaccinated* parameter sets the number of people vaccinated against the said disease prior to the outbreak. Vaccinated individuals are capable of resisting infection and can help decrease the spread of diseases by raising awareness about vaccines in their community. Parameter takes values between 1 to 100.
- The parameter for *virus spread chance* denotes the percentage probability of transmission from an infected person to a susceptible person. Higher probability leads to faster disease spread among the population. Parameter takes values between 0 to 1.
- The *recovery time* denotes the duration (in days) it takes for an infected person to recuperate and cease to spread the disease. Parameter takes values between 1 to 100.
- The *vaccine time* parameter specifies the length of time (in days) that a vaccinated person is safeguarded against contracting the disease. Parameter takes values between 0 to 2,100.
- The *vaccine costs* parameter determines the expenses incurred in inoculating an individual against the disease, including financial, time, or other elements that factor into individuals' willingness to get vaccinated. Costs are given as dimensionless ratios ranging from 1 to 10.
- The *illness-costs* parameter is determined by expenses linked with the treatment and care of the infected individual. Such expenses comprise of medical care, hospitalization, isolation, and other factors connected to the course and treatment of the disease. Costs are given as dimensionless ratios ranging from 1 to 10.

The model enables the tracking of infection rates, player strategy adoption, and maximum player payoffs across all three states. Each player operates under specific parameters that dictate its current state. The players are placed within a square grid, with each player having a maximum of eight neighboring players. The simulation involves 1,681 players and continues for 2,000 rounds, or until the epidemic has been eradicated. A subsequent version of the model also monitors mortality rates, reinfection rates, and vaccination rates. The players' decision to alter their strategy reflects the number of deaths. The strategy-selection procedure determines the players' strategy. Initially, the infection-risk parameter is calculated, indicating the risk of infection. The original calculation (8), from (Liu & Shang, 2020), appears as follows:

$$\lambda_i = \beta * \frac{N_i^{non}}{N_i^{vac} + N_i^{non}} \tag{8}$$

where λ_i is the risk of infection, β the transmission rate, N_i^{non} the number of unvaccinated neighbors and N_i^{vac} the number of vaccinated neighbours. To address the gravity of the situation, relevant data on the quantity of affected neighbours has been incorporated into the computation, resulting in the following equation (9):

$$\lambda_i^* = (\beta + \frac{N_i^{noninf}}{N_i^{inf} + N_i^{noninf}}) * \frac{N_i^{non}}{N_i^{vac} + N_i^{non}} \tag{9}$$

where N_i^{noninf} is the number of healthy neighbours and N_i^{inf} is the number of infected neighbours. Next, the cost of changing strategy, i.e., vaccinating and the cost of not vaccinating, are calculated by equation (10) and (11).

$$cost - change = (1 + \sigma_i) * r_c \tag{10}$$

$$cost - unchanged = (1 - \sigma_i) * \lambda_i \tag{11}$$

where σi is a decision value of 1 for vaccination, -1 for refusal, r_c is the cost ratio. Based on the comparison, the player decides whether or not it is good for him to be vaccinated in a given situation.

3. Results

Three experiments were conducted to demonstrate the benefits of an agent-based model as a tool for capturing the impacts of individuals' decision making in epidemic situations. The experiments aimed to address three questions related to the model.

- *Question 1:* Do the costs of illness and vaccination influence players' decision-making?
- *Question 2:* Is there a group of players who may decide against getting vaccinated during the game due to the associated cost?
- *Question 3:* Is there a correlation between the number of vaccinated players and the transmission of infection through sweat?

3.1. Question 1 – Experiment 1

The initial study aimed to examine whether lowering the vaccine/disease cost ratio, by reducing the expense of vaccination, positively impacts the population's vaccination coverage. The simulation findings support the hypothesis that individuals are inclined to vaccinate more frequently when the costs of vaccination are lower. In certain instances, the vaccination rate is either equivalent or higher when the percentage of infectivity is lower than the simulation with a greater percentage of infectivity.

It should be noted that the simulation results support the hypothesis, which leads to players opting for a more frequent vaccination strategy in circumstances where the cost is lower. However, in certain instances, the vaccination rate appears to be higher or equivalent for simulations with lower infectivity percentages, as opposed to those with higher rates. This phenomenon may arise from the comparison of the average number of vaccinated players, coupled with the limited number of simulation repetitions. In addition, the duration of the games also reflected the number of vaccinated players.

Table 1. Data from the experiment

	Percentage chances of infection									
	0.7		0.6		0.5		0.4		0.3	
Cost ratio	Number of vaccinated	Vaccinated %	Number of vaccinated	Vaccinated %	Number of vaccinated	Vaccinated %	Number of vaccinated	Vaccinated %	Number of vaccinated	Vaccinated %
1	315	0.19	317	0.19	312	0.18	316	0.19	305	0.18
0.8	494	0.29	493	0.29	491	0.29	485	0.29	415	0.24
0.6	683	0.41	681	0.41	672	0.40	677	0.40	570	0.34
0.4	826	0.49	805	0.48	786	0.48	745	0.44	646	0.38
0.2	990	0.59	869	0.51	845	0.50	777	0.46	742	0.44

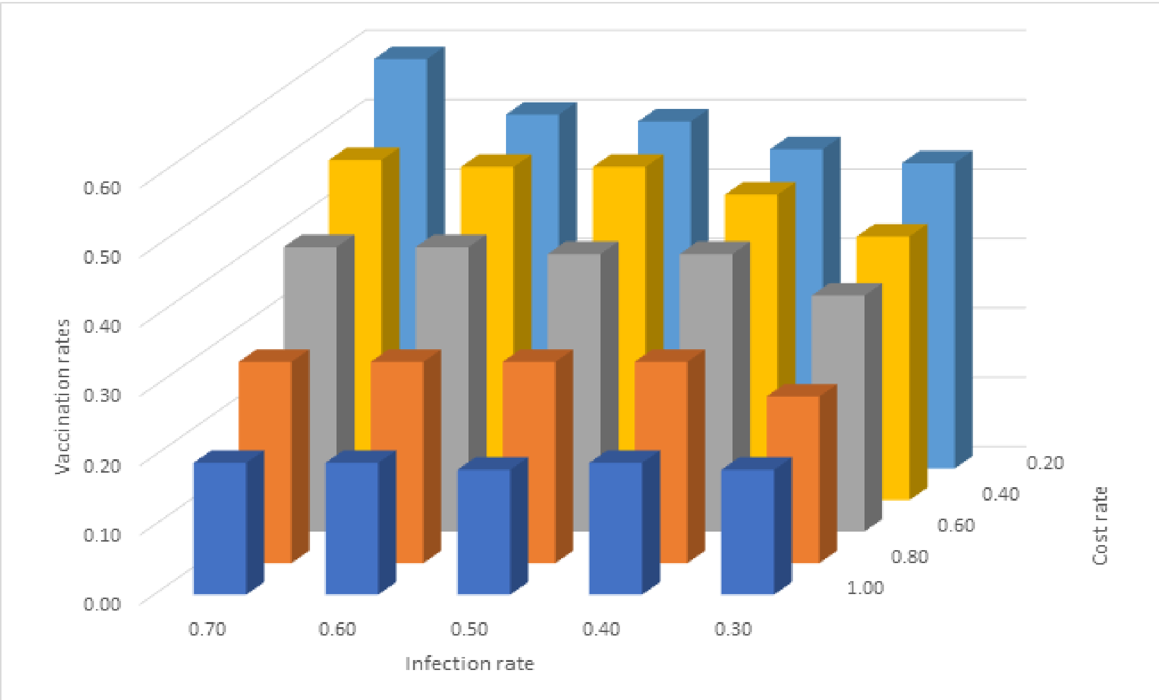

Figure 4. Dependence of the cost ratio on the percentage infection rate

The 3D plot (see Figure 4) shows the dependence of the number of vaccinated on the vaccination cost/vaccination ratios for different percentage chances of being infected. This experiment verified results article (Liu & Shang, 2020) and the values of the experiment outputs in Table 1.

3.2. Question 2 – Experiment 2

In the experiment, players will be given lifetime protection by vaccination, so at the end of the game players will remain in the susceptible and infected states, i.e. players who have not been vaccinated throughout the game. Two scenarios will be played – 2,000 rounds and 364 rounds. The cost settings are as follows:

$$r_{vac} \ll r_{inf}$$

$$r_{vac} > r_{inf}$$

where r_{vac} are vaccination costs r_{inf} are infection costs. Setting the vaccine cost much lower than the disease cost, the results for 2,000 and 364 days came out similarly: 25% and 27%. Setting the disease cost much lower than the vaccine cost, the simulations for 2,000 and 364 days came out similarly: 82% and 85%. The simulations with much lower vaccine costs approach values around 20%, but the second version of the simulation does not. This experiment verified results from (Bhattacharyya & Bauch, 2011).

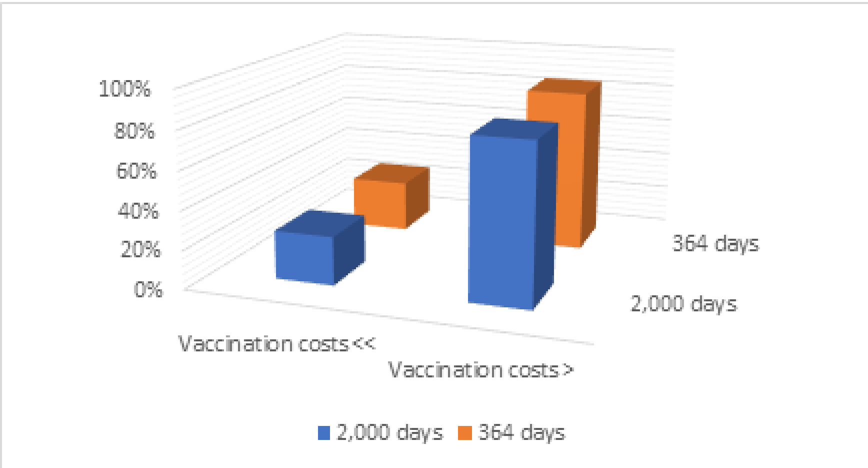

Figure 5. Nonvaccinated players

Are there unvaccinated players who are not impacted by its cost during gameplay? Results for both 2,000 and 364 days were similar at 25% and 27%, respectively, when setting vaccine cost lower than disease cost. Likewise, results for both 2,000 and 364 days were similar at 82% and 85%, respectively, when setting disease cost lower than vaccine cost in the simulation. Likewise, results for both 2,000 and 364 days were similar at 82% and 85%, respectively, when setting disease cost lower than vaccine cost in the simulation. The simulations with significantly reduced vaccine costs converge to values approximately at 20%, whereas the second version of the simulation does not display the same trend. The graphic representation shows Figure 5.

3.3. Question 3 - Experiment 3

For our experiment, we have established that the cost of illness is equal to the cost of vaccination in the model. Final protection will be given through vaccination since more players can be vulnerable to the disease when the protection expires. The length of protection is set to 180 days. Our experiment has proven that the number of vaccinated individuals fluctuates cyclically, depending on the number of infected individuals. The rate of contagion influences the initial outbreak of the epidemic and determines the maximum number of players who can be vaccinated. It also has a minor impact on the amplitude of the player count. Moreover, the risk of infection grows along with the number of infected individuals. As a result, the cost of illness escalates, which pressures the players to adopt a strategy of not getting vaccinated. Increasing the number of vaccinated players decreases vaccination costs and prevents the spread of the virus. These findings align with those of Bhattacharyya and Bauch (2010). The blue line reflects a contagion rate of 0.5, orange at 0.6, and grey at 0.7.

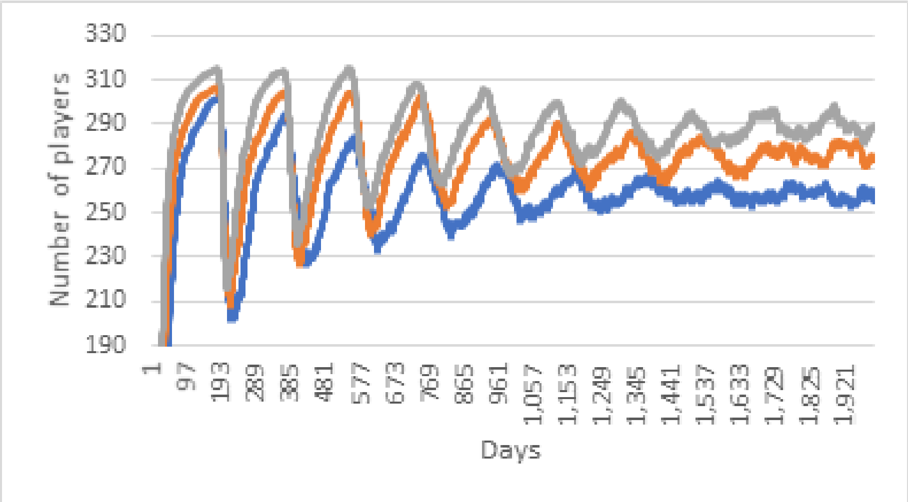

Figure 6. Number of vaccinated players

The infectivity rate has a significant impact on the initial outbreak of the epidemic and determines the maximum number of vaccinated players. It also has a slight influence on the amplitude of the player count. At the initial stage, there is a larger amplitude of fluctuation, which gradually decreases over time. However, the number of vaccinations does not fully stabilize the situation. As the number of infected individuals rises, the risk of infection also increases, leading to higher costs of illness. This compels players to opt for the non-vaccination strategy. The study confirmed that the number of vaccination strategies selected depends on the current number of infected players and follows a cyclical pattern (see Figure 6).

5. Conclusions

Based on available information, researchers developed a NetLogo model that merges epidemiology and game theory. Technical term abbreviations are explained when first used. The model simulates a symmetric and iterative vaccination game with imperfect information, where players select a strategy based on a cost comparison between disease and vaccination. Citations adhere to a consistent style guide, and quotes are clearly marked. The epidemic course and neighboring players' decisions affect the cost. The text uses passive tone and formal register, with precise word choice to ensure grammatical correctness. Players choose between two strategies: vaccination or no vaccination. The model permits the simulation of various scenarios and diseases, as input parameters are insufficiently controlled. This enables the tracking of the progression of infection levels, number of players following a specific strategy, and maximum payouts in all three states. For model validation, three experiments were conducted with published scientific articles as a reference. The results of two of these experiments were consistent with our expectations. The model rejects the use of game theory in epidemiology by simulating individuals' decision making according to the disease progression. It has the potential to expand to monitor reinfections and revaccinations and consider disease severity when determining the subjective cost and its impact on individuals' decision making.

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Conflict of interest: none.

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