

Optimal Control of Technology Development Level on Primary Energy Consumption

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Abstract: Energy is a driving force of economic growth. However, the use of energy causes lots of pollution. Technology development is strongly encouraged to promote the economic growth as well as energy conservation. In this paper, we formulate an optimal control model to minimize the total primary energy conservation in a time period. The optimal technology development level path and the primary energy consumption path is studied. Our main finding is that, to achieve an efficient economic growth path, the elasticity coefficient of GDP with respect to the primary energy consumption should be small, and the technology development investment should be high enough. In other words, high-technology industries other than heavy industries should be invested with more money to achieve a high-quality economic growth. A big elasticity coefficient of GDP with respect to the primary energy consumption will result in an inefficient development because the primary energy consumption can go higher with time.

Keywords: primary energy consumption; technology development level; optimal control theory

JEL Classification: O13; P28; Q43

1. Introduction

Energy is an important supporting material for the economic and social development of all countries in the world, and is the main driving force of economic growth (Stern, 2000). In the 40 years of reform and opening up, China's economy has developed rapidly and the demand for energy is also increasing. Now all countries in the world are promoting low-carbon economy, energy conservation and emission reduction. In China, according to the Outline of the People's Republic of China 14th Five-Year Plan (2021-2025) for National Economic and Social Development and Long-Range Objectives for 2035, the allocation and utilization of energy resources should be more efficient. The energy consumption and carbon dioxide emission per unit of GDP should be reduced by 13.5% and 18% respectively.

Previous studies have extensively examined the impact factors of energy consumption. Rahman et al. (2020) as well as Tang et al. (2016) showed that there is a unidirectional correlation from energy demand to GDP growth. Ahmad et al. (2016) showed there is a correlation from GDP growth to energy use. Tugcu and Topcu (2018) suggested that there is a bidirectional relationship between these two. Technological innovation is another factor that affects energy consumption. Wang and Wang (2020) discovered that technological

innovation has a positive impact on energy efficiency. Cao et al. (2020) found that technological improvement and use of resources are the main methods to encourage economic growth. Li and Solaymani (2021) argued that technological innovation that enhances energy efficiency is only effective in reducing energy consumption in the industrial sector.

Although many studies have already showed the relationships of energy consumption, GDP and technology development, little attention is paid to forming a theoretical framework. Many of the existing works are empirical research. In this paper, we want to study the relations of the optimal primary energy consumption path and the optimal technology development level path in a theoretical approach. We form an optimal control model to minimize the total primary energy consumption in a time period. Then we study the optimal solution paths to the problem.

This paper shows that, to achieve an efficient development path, the elasticity coefficient of GDP with respect to the primary energy consumption should be small, and the technology development investment should be high enough. A big elasticity coefficient of GDP with respect to the primary energy consumption will result in an inefficient development because the primary energy consumption can go higher with time.

This paper also shows that, we should invest on high-technology industries as much as we can to achieve higher GDP and lower primary energy consumption. For heavy industries, the investment should keep low to avoid the energy consumption.

The remaining sections are organized as follows. In Section 2 we formulate the optimal control model. In Section 3 we calculate the model to achieve the optimal paths of technology development level, GDP and primary energy consumption. Some further analyses are also implemented. Section 4 concludes the paper and suggests directions for future research.

2. The Model

In this section, we consider a dynamic optimization problem in a given time interval to minimize the primary energy consumption. We use an optimal control model to find the optimal path of technology development level.

The IPAT model (Ehrlich & Holdren, 1971) has been widely used to study the impact of population (P), affluence (A) and technology (T) on the environment, for example, CO₂ emissions (see, e.g., Soulé & DeHart, 1998; Chontanawat, 2018). In the field of this method, affluence is often replaced with per capita GDP. Extensive studies showed that energy consumption has a positive and significant effect on CO₂ emissions (see, e.g., Boutabba, 2014; Heidari et al., 2015). Thus, we use the primary energy consumption to replace the environmental impact and taking economic growth, technology development and population size into consideration, we can form the IPAT model as below.

$$E(t) = aA(t)^{-1}G(t)^\alpha L(t)^\beta, \quad (1)$$

where $E(t)$, $A(t)$, $G(t)$ and $L(t)$ represent the primary energy consumption, technology development level, GDP and population respectively in time t . Here α and β are elasticity coefficients of GDP and population respect to the primary energy consumption, and a is a positive constant.

The production function is assumed to have a Cobb-Douglas form:

$$G(t) = bA(t)K(t)^\mu L(t)^\eta, \quad (2)$$

where $K(t)$ stands for the capital amount. Here μ and η are elasticity coefficients of capital amount and population respect to GDP, and b is a positive constant.

Replacing $G(t)$ in Equation (1) by its Cobb-Douglas form in Equation (2), we have:

$$E(t) = ab^\alpha A(t)^{\alpha-1} K(t)^{\alpha\mu} L(t)^{\alpha\eta+\beta}. \quad (3)$$

Technology development level $A(t)$ is the cumulative level of long-run technological changes. Denote $I(t)$ as the capital amount in technology development. We assume that $A(t)$ linearly changes with $I(t)$:

$$A(t) = gI(t) + g_0, \quad (4)$$

where $g > 0$ represents the contribution parameter of the capital amount in technology development to the technology development level, and g_0 is a positive constant. We further assume that $I(t)$ satisfies:

$$\dot{I}(t) = u(t) - \delta I(t), \quad (5)$$

where $u(t)$ stands for the technology development investment rate. We assume that the technology development investment rate has a boundary, i.e., $u(t) \in [\underline{u}, \bar{u}]$. Suppose that both \underline{u} and \bar{u} are positive. And δ is the depreciation rate of the capital amount in technology development.

The objective is to minimize the total primary energy consumption in a given time interval $[0, T]$. The technology development level $A(t)$ is the state variable. Whereas the technology development investment rate $u(t)$ is the control variable. Without loss of generality, we further assume that the technology development level in time 0 is positive and is known as $A(0) = A_0$, but $A(T)$ can be free. The optimal control problem is then formulated as:

$$\begin{aligned} \min \quad & V[A(t)] = \int_0^T E(t) dt = \int_0^T ab^\alpha A(t)^{\alpha-1} K(t)^{\alpha\mu} L(t)^{\alpha\eta+\beta} dt \\ \text{subject to} \quad & A(t) = gI(t) + g_0, \\ & \dot{I}(t) = u(t) - \delta I(t), \\ & A(0) = A_0, \quad A(T) \text{ free}, \\ \text{and} \quad & u(t) \in [\underline{u}, \bar{u}]. \end{aligned} \quad (6)$$

3. Results

In this section we find the optimal path of the technology development level $A(t)$ and the technology development investment rate $u(t)$. Then we calculate the optimal paths of technology development level, GDP and primary energy consumption. Condition for an efficient economic development is given at the end.

3.1. Maximizing the Hamiltonian

Let us first change the objective into a maximize form:

$$\max -V[A(t)] = - \int_0^T E(t) dt, \quad (7)$$

then the Hamiltonian is:

$$H = -E(t) + \lambda(t)(u(t) - \delta I(t)), \quad (8)$$

where $\lambda(t)$ is the costate variable. We replace the state variable $A(t)$ with Equation (4), then the Hamiltonian becomes:

$$H = -ab^\alpha(gI(t) + g_0)^{\alpha-1}K(t)^{\alpha\mu}L(t)^{\alpha\eta+\beta} + \lambda(t)(u(t) - \delta I(t)). \quad (9)$$

Noticing that H is linear in the control variable $u(t)$ with slope $\frac{\partial H}{\partial u(t)} = \lambda(t)$, and $u(t)$ has a boundary $[\underline{u}, \bar{u}]$, to maximize H with respect to $u(t)$, we have to choose $u^*(t) = \bar{u}$ if $\lambda(t) > 0$, and $u^*(t) = \underline{u}$, if $\lambda(t) < 0$. In short,

$$u^*(t) = \begin{cases} \bar{u} & \text{if } \lambda(t) > 0, \\ \underline{u} & \text{if } \lambda(t) < 0. \end{cases} \quad (10)$$

3.2. The Optimal Costate Path, Control Path and State Path

The search for the costate path begins with the equation of motion

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial I(t)} = ab^\alpha g(\alpha - 1)(gI(t) + g_0)^{\alpha-2}K(t)^{\alpha\mu}L(t)^{\alpha\eta+\beta} + \delta\lambda(t). \quad (11)$$

The equation is a first-order linear differential equation with a constant coefficient but a variable term.

The right-hand side of Equation (11) has an unknown variable $I(t)$. Since we only care if $\lambda(t)$ is positive or negative, we now split the problem into two cases, namely $\lambda(t) > 0$ and $\lambda(t) < 0$.

Case 1. If $\lambda(t) > 0$, by using Equation (10), we have $u^*(t) = \bar{u}$. Substituting $u^*(t) = \bar{u}$ into Equation (5) yields

$$\dot{I}(t) = \bar{u} - \delta I(t). \quad (12)$$

It follows that the general solution for $I(t)$ in this first-order differential equation is

$$I(t) = c_1 e^{-\delta t} + \frac{\bar{u}}{\delta}, \quad (13)$$

where c_1 is an arbitrary constant to be definitized. Substituting Equation (13) into (4), by using the initial condition $A(0) = A_0$, we have

$$A^*(t) = \left(A_0 - g_0 - \frac{g\bar{u}}{\delta} \right) e^{-\delta t} + \left(\frac{g\bar{u}}{\delta} + g_0 \right). \quad (14)$$

By using Equation (4) again we have $c_1 = \frac{A_0 - g_0}{g} - \frac{\bar{u}}{\delta}$, and

$$I^*(t) = \left(\frac{A_0 - g_0}{g} - \frac{\bar{u}}{\delta} \right) e^{-\delta t} + \frac{\bar{u}}{\delta}. \quad (15)$$

Substituting Equation (15) into (11), we have

$$\dot{\lambda}(t) = ab^\alpha g(\alpha - 1) \left(\left(A_0 - g_0 - \frac{g\bar{u}}{\delta} \right) e^{-\delta t} + \left(\frac{g\bar{u}}{\delta} + g_0 \right) \right)^{\alpha-2} K(t)^{\alpha\mu} L(t)^{\alpha\eta+\beta} + \delta\lambda(t). \quad (16)$$

The general solution for $\lambda(t)$ can then be derived as

$$\lambda(t) = e^{\delta t} \left[\int Q(t) e^{-\delta t} dt + c_2 \right], \quad (17)$$

where $Q(t) = ab^\alpha g(\alpha - 1) \left(\left(A_0 - g_0 - \frac{g\bar{u}}{\delta} \right) e^{-\delta t} + \left(\frac{g\bar{u}}{\delta} + g_0 \right) \right)^{\alpha-2} K(t)^{\alpha\mu} L(t)^{\alpha\eta+\beta}$, and c_2 is an arbitrary constant to be definitized.

To definitize c_2 , we can make use of the transversality condition for the vertical-terminal-line optimal control problem, $\lambda(T) = 0$. Letting $t = T$ in Equation (17), applying the transversality condition, we find that $c_2 = \left[-\int Q(t) e^{-\delta t} dt \right]_{t=T}$.

Case 2. If $\lambda(t) < 0$, by using Equation (10), we have $u^*(t) = \underline{u}$. Similar to the approach we used in Case 1, we can derive the following results. The general solution for $I(t)$ is

$$I(t) = c_1 e^{-\delta t} + \frac{u}{\delta}. \quad (18)$$

The optimal state path for $A(t)$ is

$$A^*(t) = \left(A_0 - g_0 - \frac{g\underline{u}}{\delta} \right) e^{-\delta t} + \left(\frac{g\underline{u}}{\delta} + g_0 \right). \quad (19)$$

And the optimal path for $I(t)$ is

$$I^*(t) = \left(\frac{A_0 - g_0 - \frac{u}{\delta}}{g} \right) e^{-\delta t} + \frac{u}{\delta}. \quad (20)$$

The optimal path for $\lambda(t)$ becomes

$$\lambda(t) = e^{\delta t} \left[\int Q(t) e^{-\delta t} dt - \left[\int Q(t) e^{-\delta t} dt \right]_{t=T} \right], \quad (21)$$

where $Q(t) = ab^\alpha g(\alpha - 1) \left(\left(A_0 - g_0 - \frac{g\underline{u}}{\delta} \right) e^{-\delta t} + \left(\frac{g\underline{u}}{\delta} + g_0 \right) \right)^{\alpha-2} K(t)^{\alpha\mu} L(t)^{\alpha\eta+\beta}$.

In the end of this section, we give a sufficient condition under which $\lambda(t)$ can be positive and negative.

Lemma 1. If the elasticity coefficient of GDP with respect to the primary energy consumption $\alpha < 1$, then $\lambda(t) > 0$ for all $t \in [0, T]$. If the elasticity coefficient of GDP with respect to the primary energy consumption $\alpha > 1$, then $\lambda(t) < 0$ for all $t \in [0, T]$.

Proof. From Equations (17) and (21), we know $\lambda(t) = e^{\delta t} \left[\int Q(t) e^{-\delta t} dt - \left[\int Q(t) e^{-\delta t} dt \right]_{t=T} \right]$,

where $Q(t) = ab^\alpha g(\alpha - 1) \left(\left(A_0 - g_0 - \frac{g u^*}{\delta} \right) e^{-\delta t} + \left(\frac{g u^*}{\delta} + g_0 \right) \right)^{\alpha-2} K(t)^{\alpha\mu} L(t)^{\alpha\eta+\beta}$. Here u^* is either \bar{u} if $\lambda(t) > 0$ or \underline{u} if $\lambda(t) < 0$. For both $u^* = \bar{u}$ and $u^* = \underline{u}$, we can derive that $\left(A_0 - g_0 - \frac{g u^*}{\delta} \right) e^{-\delta t} + \left(\frac{g u^*}{\delta} + g_0 \right) = A_0 e^{-\delta t} + (1 - e^{-\delta t}) \left(\frac{g u^*}{\delta} + g_0 \right) > 0$ holds for all $t \in [0, T]$. Thus $ab^\alpha g \left(\left(A_0 - g_0 - \frac{g u^*}{\delta} \right) e^{-\delta t} + \left(\frac{g u^*}{\delta} + g_0 \right) \right)^{\alpha-2} K(t)^{\alpha\mu} L(t)^{\alpha\eta+\beta} > 0$ holds.

If $\alpha < 1$, then we have $Q(t) < 0$ as well as $Q(t)e^{-\delta t} < 0$ for all $t \in [0, T]$. Noticing that $Q(t)e^{-\delta t}$ is the derivative of $\int Q(t)e^{-\delta t} dt$, we know that $\int Q(t)e^{-\delta t} dt$ is a decreasing function in t . Thus $\int Q(t)e^{-\delta t} dt - [\int Q(t)e^{-\delta t} dt]_{t=T} > 0$ holds for all $t \in [0, T]$, which give us the result that $\lambda(t) > 0$ for all $t \in [0, T]$.

Similarly, if $\alpha > 1$, we have $Q(t) > 0$ as well as $Q(t)e^{-\delta t} > 0$ for all $t \in [0, T]$. Thus $\int Q(t)e^{-\delta t} dt$ is an increasing function in t . Thus $\int Q(t)e^{-\delta t} dt - [\int Q(t)e^{-\delta t} dt]_{t=T} < 0$ holds for all $t \in [0, T]$, which give us the result that $\lambda(t) < 0$ for all $t \in [0, T]$. This ends the proof of Lemma 1.

3.3. Analysis of the Optimal Paths of Technology Development Level, Economic Growth and Primary Energy Consumption

From the results showed in Section 3.2, we analyze the optimal paths of technology development level $A^*(t)$, economic growth level $G^*(t)$ and the primary energy consumption $E^*(t)$ in two cases, namely $\alpha < 1$ and $\alpha > 1$.

Case 1. The elasticity coefficient of GDP with respect to the primary energy consumption $\alpha < 1$.

From Lemma 1, we know that $\lambda(t) > 0$ for all $t \in [0, T]$. Taking Equation (10) into consideration, the optimal technology development investment rate $u(t)$ should always keep its maximum value \bar{u} . According to Equation (14), the optimal path of technology development level is $A^*(t) = (A_0 - g_0 - \frac{g\bar{u}}{\delta})e^{-\delta t} + (\frac{g\bar{u}}{\delta} + g_0)$. As discussed in Section 3.2, $A^*(t)$ is always positive. However, if the technology development investment rate \bar{u} is high enough, i.e., $A_0 - g_0 - \frac{g\bar{u}}{\delta} < 0$, then the optimal technology development level $A^*(t)$ will be an exponentially increasing function with time t . Substituting $A^*(t)$ into Equations (2) and (3), it turns out that the corresponding economic growth path $G^*(t)$ will increase in time, whereas the corresponding primary energy consumption $E^*(t)$ will decrease in time. This is an efficient development case. We put these results in Theorem 1 given below.

Theorem 1. (Efficient development case) When the elasticity coefficient of GDP with respect to the primary energy consumption $\alpha < 1$, the optimal technology development investment rate is $u^*(t) = \bar{u}$. If \bar{u} satisfies that $\bar{u} > \frac{\delta}{g}(A_0 - g_0)$, then the optimal technology development level $A^*(t)$ and the optimal economic growth $G^*(t)$ will increase in time. The optimal primary energy consumption $E^*(t)$ is decrease in time.

By the similar approach, we have results if \bar{u} is not high enough, i.e., $\bar{u} < \frac{\delta}{g}(A_0 - g_0)$. We list it in Theorem 2.

Theorem 2. (Inefficient development case) When the elasticity coefficient of GDP with respect to the primary energy consumption $\alpha < 1$, the optimal technology development investment rate is $u^*(t) = \bar{u}$. If \bar{u} satisfies that $\bar{u} < \frac{\delta}{g}(A_0 - g_0)$, then the optimal technology development level $A^*(t)$ and the optimal economic growth $G^*(t)$ will decrease in time. The optimal primary energy consumption $E^*(t)$ is increase in time.

Case 2. The elasticity coefficient of GDP with respect to the primary energy consumption $\alpha > 1$.

From Lemma 1, we know that $\lambda(t) < 0$ for all $t \in [0, T)$. Taking Equation (10) into consideration, the optimal technology development investment rate $u(t)$ should always keep its minimum value \underline{u} . In this case, similar to Case 1, we can derive that $A^*(t)$ is always positive. Furthermore, if $A_0 - g_0 - \frac{gu}{\delta} < 0$, we can derive that $A^*(t)$, $G^*(t)$, and $E^*(t)$ are all increase in time. If $A_0 - g_0 - \frac{gu}{\delta} > 0$, we have $A^*(t)$, $G^*(t)$, and $E^*(t)$ are all decrease in time. We conclude it formally in Theorems 3 and 4.

Theorem 3. (Inefficient development case) When the elasticity coefficient of GDP with respect to the primary energy consumption $\alpha > 1$, the optimal technology development investment rate is $u^*(t) = \underline{u}$. If \underline{u} satisfies that $\underline{u} > \frac{\delta}{g}(A_0 - g_0)$, then the optimal technology development level $A^*(t)$, the optimal economic growth $G^*(t)$ and the optimal primary energy consumption $E^*(t)$ will all increase in time.

Theorem 4. (Inefficient development case) When the elasticity coefficient of GDP with respect to the primary energy consumption $\alpha > 1$, the optimal technology development investment rate is $u^*(t) = \underline{u}$. If \underline{u} satisfies that $\underline{u} < \frac{\delta}{g}(A_0 - g_0)$, then the optimal technology development level $A^*(t)$, the optimal economic growth $G^*(t)$ and the optimal primary energy consumption $E^*(t)$ will all decrease in time.

Here we label Theorems 2-4 all with inefficient cases because either $G^*(t)$ is decrease in time or $E^*(t)$ is increase in time. An increase of $E^*(t)$ shows the primary consumption is going up, and a decrease of $G^*(t)$ shows that the economic growth is slowing down.

In the end of this section, we put all the results into Table 1 below.

Table 1. The monotonicity of the optimal paths of $(A^*(t), G^*(t), E^*(t))$

	$u^* > \frac{\delta}{g}(A_0 - g_0)$	$u^* < \frac{\delta}{g}(A_0 - g_0)$
$\alpha < 1 (u^* = \bar{u})$	(+,+,-) Efficient	(-,+) Inefficient
$\alpha > 1 (u^* = \underline{u})$	(+,+,+) Inefficient	(-,-,-) Inefficient

Note: The symbol "+" means the optimal path is increasing in time, and "-" means the optimal path is decreasing in time. We say the case is efficient if $G^*(t)$ is increasing in time and $E^*(t)$ is decreasing in time. Either a decreasing $G^*(t)$ or an increasing $E^*(t)$ is considered as inefficient.

As shown in Table 1, to achieve an efficient development path, the elasticity coefficient of GDP with respect to the primary energy consumption α should be small, and the technology development investment should be high enough. What we should notice is that, even if the technology development investment is high enough, a big elasticity coefficient of GDP with respect to the primary energy consumption α still results an inefficient development because the primary energy consumption can go higher with time.

In the meanwhile, let's take a deeper look at the elasticity coefficient α . If α is small, it means that the increasing rate of primary energy consumption is less related to the increasing rate of GDP. High-technology industries with less energy consumption would be the main contribution to GDP. In this case, our results show that the technology development investment should be its upper bound. The more investment they have, the more rapid GDP goes higher and less energy consumption is needed. In the contrary, a large α means that

the increasing rate of primary energy consumption is closely related to the increasing rate of GDP. Heavy industries would be the main contribution to GDP. In this case, our results show that the technology development investment should be its lower bound. Even the GDP still goes up with more technology development investment, it would cause a huge energy consumption.

4. Discussion

This paper builds an optimal control model to minimize the total primary energy consumption in a time period. The relations of the optimal primary energy consumption path and the optimal technology development level are revealed in a theoretical approach.

Our main findings are listed below. First, to achieve an efficient development path, the elasticity coefficient of GDP with respect to the primary energy consumption should be small, and the technology development investment should be high enough. Second, A big elasticity coefficient of GDP with respect to the primary energy consumption will results an inefficient development because the primary energy consumption can go higher with time. Third, high-technology industries other than heavy industries should be invested with more money to promote economic growth as well as energy conservation.

This paper can be extended in several ways. First, in this paper, we only take into one state variable $A(t)$. However, the capital amount $K(t)$ is also a driving factor for both economic growth and energy conservation. An analysis with both $A(t)$ and $K(t)$ might give more fruitful results. Second, empirical research with data in China can also be added. These extensions are left to be future research.

Conflict of interest: none

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