

Searching Efficient Project Portfolios Using Data Envelopment Analysis

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Abstract: Project management is a discipline for effective resource management in order to meet the specific goals of the organization. Organizations usually implement several projects at once, which form a project portfolio. In time, possible projects come in organizations, which are characterized by the required resources and the achievement of the organization's goals. The required resources form the inputs and the achieved level of fulfillment of the organization's goals forms the outputs of the created model. Efficient projects can be then determined using the basic data envelopment analysis (DEA) method. Total resources are available in the organization in limited amounts. A portfolio composed only of effective individual projects may not always provide an effective portfolio due to maximizing overall output with limited resources. The article suggests a new method for project portfolio forming based on a generalized DEA model. The efficiency of the portfolio and the effectiveness of outputs are used as performance measures of the designed project portfolio. Useful extensions of the procedure are considered and analyzed.

Keywords: project portfolios; efficiency; DEA; resources; goals

JEL Classification: C44

1. Introduction

Project management is a tool for successful promotion of the organization's goals by planning and organizing a set of interconnected activities, the implementation of which is ensured by drawing on available resources. Nowadays, the principles of project management are increasingly used as the life of products is shortened. Everything changes, and this creates pressure on organizations to complete new products faster, less expensive and with low risk. There is a very extensive literature on the management of individual projects and project portfolios (see e.g. Kerzner, 2013; Turner, 2016). Project management can be described as a managerial process (Larson & Gray, 2013).

A multi-project environment is typical for project organizations. Organizations need to address the interdependence of projects and the sharing of common resources over time. Projects are tools for enforcing an organization's strategy. Strategic alignment of projects has a major impact on the efficient use of resources to achieve the goals of the organization. Project portfolios must be formed to contribute to this as a whole.

The first Data Envelopment Analysis (DEA) model was developed by Charnes, Cooper, and Rhodes (Charnes et al., 1978). The DEA consist of a number of models and methods to performance evaluation of units (Cooper et al., 2006; Charnes et al., 2013). The DEA model uses aggregation by weights of multiple inputs and multiple outputs into one combined input and one combined output. The DEA can be taken as a method of precise determination of weights.

The aim of the paper is to propose a procedure for searching effective projects and project portfolios. The DEA can be used to determine efficient projects, where possible projects are determined using inputs and outputs. The inputs capture the required resource levels for project implementation. Outputs measure the achieved levels of goals using multiple criteria. A portfolio composed only of effective individual projects may not always provide an effective portfolio due to maximizing overall output with limited resources. The article suggests a new method for project portfolio forming based on a generalized DEA model. Portfolios are considered as a combined project consisting of a selection of possible individual projects. The model calculates weights that maximize the efficiency of the entire project portfolio.

The article is arranged as follows. Section 2 deals with the formulation of the project portfolio problem. Searching efficient projects using the DEA method is described in Section 3. Section 4 provides a generalized DEA model for searching efficient project portfolios. A simple example is solved in Section 5. Discussion and conclusion are given in Section 6.

2. Project Portfolio Management

Project portfolio management is dedicated to the organization and management of all projects of the organization with regard to the coordination of projects, sharing common resources for projects, up to the level of individual projects (Enoch, 2015; Levine, 2005). The project office is a project portfolio management department that connects strategic management, through tactical management, to the level of operational project management.

Potential projects appear over time and it is necessary to consider whether they are suitable for inclusion in the project portfolio or not (Figure 1).

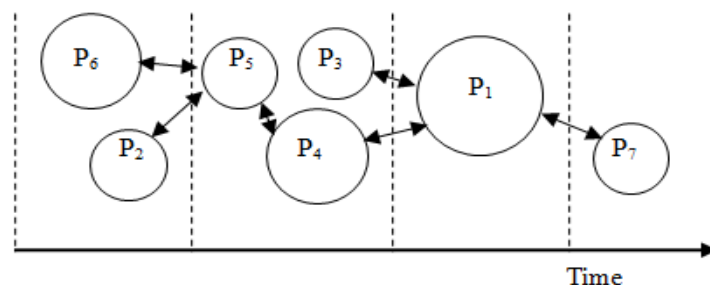


Figure 1. Dynamic flow of projects

The dynamic flow of projects (Figure 1) significantly affects the overall efficiency of project portfolios. The project portfolio consists of a group of projects that are evaluated in the organization at a given time. It is also necessary to consider whether projects that are already

in the portfolio that are not effective need to be removed from the portfolio. The main goals of the project portfolio management contain:

- Monitoring the achievement of the organization's strategic goals,
- Total project portfolio optimization and not sub-projects,
- Identification of starting projects,
- Rejection of inappropriate projects,
- Interruption or termination of ongoing inappropriate projects,
- Setting project priorities,
- Resource sharing and coordination.

Measuring the fulfillment of the organization's strategic goals is associated with the use of multi-criteria evaluation procedures. Because the main goal is to increase the value of the organization, financial criteria are mainly used (e.g. net present value, internal rate of return, etc.) Of course, in addition to the financial criteria, other criteria should also be considered for the effective evaluation of project portfolios, such as:

- Goals consistency (from strategic to operational ones),
- Balancing between available resources of all kinds and their efficient use,
- Monitoring time-dependent use of resources,
- Ensuring links between projects (in terms of continuity, preferences, resource sharing),
- Optimization of project portfolios in terms of goals, time, costs, use of resources, risks.

Many researchers and managers are working in various sophisticated ways to improve project portfolio management. Less work is devoted to quantitative analysis of the efficiency of projects and project portfolios. The procedures are based on the modification of DEA models (Cook & Green, 2000; Lengacher & Cammarata, 2012). The procedure proposed in this paper looks for project portfolios with maximal efficiency ratios with restrictions given by number of projects or by maximizing overall output with limited resources.

3. Efficient Individual Projects

The organization considers the set $P = \{P_1, P_2, \dots, P_n\}$ of n potential projects, each of which needs values of r inputs and provides values of s outputs; (r, n) -matrix X and (s, n) -matrix Y are the measured input and output values, u_i, v_i are the weights of outputs and inputs for projects. Inputs are resources (human, material, financial and others) for project implementation. Outputs are the results (tangible, intangible, financial and others) provided by projects. The CCR (Charnes, Cooper, and Rhodes) model with constant return to scale is a useful tool that is also suitable for measuring the efficiency of projects. This model was chosen because it satisfies the property that a change in the amounts of inputs leads to a similar change in the amounts of outputs. The ratio of the weighted combined output to the weighted combined input is taken as an efficiency measure. The relative efficiency e_k of the project P_k is maximised with constraints that the relative efficiency of each project is less than or equal to one.

The DEA-based model evaluates the relative efficiency of individual projects with respect to all considered potential projects. This model forms the linear fractional programming problem:

$$e_k = \frac{\sum_{i=1}^s u_i y_{ik}}{\sum_{j=1}^r v_j x_{jk}} \rightarrow \max, \quad k = 1, 2, \dots, n \quad (1)$$

$$\frac{\sum_{i=1}^s u_i y_{ih}}{\sum_{j=1}^r v_j x_{jh}} \leq 1, \quad h = 1, 2, \dots, n \quad (2)$$

$$u_i, v_j \geq 0, \quad i = 1, 2, \dots, s, \quad j = 1, 2, \dots, r \quad (3)$$

A project is called efficient if there are weights for which the efficiency ratio e_k of the project is equal to one. In this way, a set of effective projects is selected. Projects with an efficiency ratio of less than one are considered inefficient.

The Charnes-Cooper transformation changes the solution of the fractional programming problem to the easy solution of the linear programming problem:

$$e_k = \sum_{i=1}^s u_i y_{ik} \rightarrow \max, \quad k = 1, 2, \dots, n, \quad (4)$$

$$\sum_{j=1}^r v_j x_{jk} = 1 \quad (5)$$

$$\sum_{i=1}^s u_i y_{ih} - \sum_{j=1}^r v_j x_{jh} \leq 0, \quad h = 1, 2, \dots, n \quad (6)$$

$$u_i, v_j \geq 0, \quad i = 1, 2, \dots, s, \quad j = 1, 2, \dots, r \quad (7)$$

The order of the projects is then given by the values of the efficiency scores e_k . A portfolio composed only of effective individual projects may not always provide an effective portfolio due to maximizing overall output with limited resources.

4. Efficient Project Portfolios

A project portfolio is defined as a subset C of the set of possible projects P ($C \subseteq P$). The portfolio is modelled as a single combined project. The combined project is given by a combination of outputs and a combination of inputs. The combinations are given by a vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$ where $\lambda_i = 1$ (the individual project P_i is a part of the portfolio) or $\lambda_i = 0$ (the individual project P_i is not a part of the portfolio). Total inputs of the combined project marked with the symbol $x_j(C) = \sum_{h=1}^n \lambda_h x_{jh}$, $j = 1, 2, \dots, r$, and total outputs marked with the symbol $y_i(C) = \sum_{h=1}^n \lambda_h y_{ih}$, $i = 1, 2, \dots, s$, are given by the combination vector λ . The set of all combined projects is the so-called power set of the set of potential projects P . The power set is marked with the symbol $R(P)$ and the number of elements in the set $R(P)$ is equal to $2^n - 1$, which is the number of nonempty subsets of a set of n elements.

The DEA method may be specified for the evaluation of each project portfolio with respect to the power set $R(P)$.

$$e_c = \sum_{i=1}^s u_i \sum_{h=1}^n \lambda_h y_{ih} \rightarrow \max \quad (8)$$

$$\sum_{j=1}^r v_j \sum_{h=1}^n \lambda_h x_{jh} = 1 \quad (9)$$

$$\sum_{i=1}^s u_i \sum_{h=1}^n \lambda_h y_{ih} - \sum_{j=1}^r v_j \sum_{h=1}^n \lambda_h x_{jh} \leq 0, \quad C \in R(P) \quad (10)$$

$$\lambda_h \in \{0, 1\}, h = 1, 2, \dots, n \quad (11)$$

$$u_i, v_j \geq 0, \quad i = 1, 2, \dots, s, \quad j = 1, 2, \dots, r \quad (12)$$

The order of the projects is then given by the values of the efficiency scores e_k . A portfolio composed only of effective individual projects may not always provide an effective portfolio due to maximizing overall output with limited resources.

That model (8) - (12) is nonlinear with variables λ_h, u_i, v_i where λ_h are the elements of the combination vector of projects and u_i, v_i are the weights of outputs and inputs for projects. However, the model is difficult to solve for the large number of constraints (10).

The introduction of new variables

$$c_{ih} = u_i \lambda_h, \quad d_{jh} = v_j \lambda_h, \quad i = 1, 2, \dots, s, \quad j = 1, 2, \dots, r, \quad h = 1, 2, \dots, n \quad (13)$$

allows to create the linear problem. Portfolios with their combined inputs and outputs are compared with the set of all $R(P)$ portfolios. Portfolio constraints (10) are only an additive combination of constraints for individual projects and therefore these constraints can be replaced by constraints (16) that only compare projects from the set P of all projects (Cook & Green, 2000).

Restrictions for portfolios that are given by a combination of projects are superfluous. The restrictions (19) and (20) define the relations between new variables c_{ih}, d_{jh} and old variables u_i, v_j, λ_h , where M is a large number. The restrictions (19) define the relationships between the variables c_{ih}, u_i, λ_h : if the binary variable $\lambda_h = 1$, then $0 \leq c_{ih} \leq M, u_i = c_{ih}$ and if the binary variable $\lambda_h = 0$, then $0 \leq u_i \leq M, c_{ih} = 0$. The restrictions (20) analogically define the relationships between the variables d_{jh}, v_j, λ_h .

The task is therefore defined as follows:

$$e_c = \sum_{i=1}^s \sum_{h=1}^n c_{ih} y_{ih} \rightarrow \max \quad (14)$$

$$\sum_{j=1}^r \sum_{h=1}^n d_{jh} x_{jh} = 1 \quad (15)$$

$$\sum_{i=1}^s u_i y_{ih} - \sum_{j=1}^r v_j x_{jh} \leq 0, \quad h = 1, 2, \dots, \quad (16)$$

$$\lambda_h \in \{0, 1\}, h = 1, 2, \dots, n \quad (17)$$

$$\sum_{i=1}^s u_i y_{ih} - \sum_{j=1}^r v_j x_{jh} \leq 0, \quad h = 1, 2, \dots, n \quad (18)$$

$$c_{ih} \geq 0, c_{ih} \leq M \lambda_h, \quad u_i \geq c_{ih}, u_i \leq c_{ih} + M(1 - \lambda_h), \quad i = 1, 2, \dots, s, h = 1, 2, \dots, n \quad (19)$$

$$d_{jh} \geq 0, d_{jh} \leq M \lambda_h, \quad v_j \geq d_{jh}, v_j \leq d_{jh} + M(1 - \lambda_h), \quad j = 1, 2, \dots, r, h = 1, 2, \dots, n \quad (20)$$

The task (14)-(20) can be used for detailed project portfolio analysis, to calculate the maximal efficiency ratios for different project portfolio structures for a specified number of projects.

5. Simple Numerical Example

We will show a simple numerical example on which we will demonstrate the proposed method for the detailed analysis of project portfolio structures for a specified number of projects. An organisation considers a set of 5 potential projects (P_1, P_2, \dots, P_5) that have two inputs ($I1, I2$) and two outputs ($O1, O2$) assigned to them. Data for the set of potential projects are presented in Table 1.

Table 1. Data for a set of potential projects.

	P_1	P_2	P_3	P_4	P_5
$I1_i$	6	3	8	9	5
$I2_i$	8	4	2	4	6
$O1_i$	9	7	6	10	8
$O2_i$	12	10	15	8	12
e_i	0.643	1	1	1	0.761

An analysis of project portfolios will be done at one specific point in time. With a dynamic flow of projects, such an analysis would be made at each time a new project arrives or an existing project ends. The solution of the model (4)-(7) provides the efficiency ratios e_i of all projects. The efficiency ratios with value one determine efficient projects. The set of efficient projects is formed by the projects P_2, P_3, P_4 . There are $2^5 - 1 = 31$ possible project portfolios, which can be organized into structures according to the number of projects, from 1-project structure (individual projects) to 5-projects (all projects) structure.

The task (14)-(20) determined efficiency ratios for project portfolios for all structures. Table 2 contains portfolio structures, a number of portfolios for a given structure, maximal efficiency ratios e_c in the structure, and the project portfolios with maximal efficiency ratios, which may not be equal to one, as it may not only contain effective projects.

Table 2. Portfolio structures

Structure	Number	Max e_c	Portfolios
1-project	5	1	P_2, P_3, P_4
2-projects	10	1	$(P_2, P_3), (P_2, P_4), (P_3, P_4)$
3-projects	10	1	(P_2, P_3, P_4)
4-projects	5	0.920	(P_2, P_3, P_4, P_5)
5-projects	1	0.838	$(P_1, P_2, P_3, P_4, P_5)$

6. Discussion and Conclusion

The article proposes a new method for finding efficient project portfolios. The method is based on the solution of a linear program with binary variables. This method is a useful tool for detailed analysis of project portfolios and brings important conclusions in comparison with

other methods. However, project portfolios are compared not only according to efficiency, but also according to the ability to implement as many projects as possible with respect to the consumption of available resources. Related to this is the applicability of multi-criteria approaches.

This method is flexible and can be used for all types of projects. It is also possible to use the proposed procedure for the analysis and search for suitable elements of other systems, such as the search for efficient supply chain links (Fiala, 2016).

This method also provides possible extensions. In the basic DEA model, the weights are not limited in any way, but it is possible to easily limit them due to preferences. Analytic Hierarchy Process (AHP) (Saaty, 1990) is a suitable tool for expressing the preferences of the decision maker using scales. The estimates are entered in the comparison matrix $C = (c_{jk})$, where elements c_{jk} are weight ratio estimates w_j/w_k .

The basic DEA model works with accurate data, but in reality, the data are imprecise. To get closer to reality, imprecise DEA with interval data for inputs and outputs can be used (Smirlis et al., 2004). The proposed method can be supplemented by other procedures, as De Novo optimization (Fiala, 2018). The search for effective project portfolios can be followed due to several criteria. Dual problem analysis can provide managers with important information for more detailed portfolio analysis (Fiala, 1981) Stakeholders with different interests follow the search for efficient project portfolios. A consensual solution can be found in the negotiation process while monitoring the values of multiple criteria (Fiala, 1999).

Possible extensions of the proposed method will be the subject of further research and we believe that their combinations will provide an even richer tool for project portfolio and other analysis.

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Conflict of interest: none

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