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Dynamic Analytic Network Process

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Abstract. The AHP (Analytic Hierarchy Process) method is modified with respect to network structures and dynamics of the analyzed systems. The ANP (Analytic Network Process) method is appropriate for setting priorities in network systems where there are different types of dependencies between evaluation criteria and system elements. However, with time-varying environments in network systems, time-dependent priorities play an increasingly important role. Long-term priorities can be based on time-dependent comparisons of criteria and system elements. For short-term prediction, exponential smoothing of compositional data can be used. The paper proposes a hybrid procedure DNAP (Dynamic Analytic Network Process) that combines and enriches advantages and benefits of both approaches by analysis of network systems.

Keywords: Analytic Network Process, Time Dependent Priorities, Compositional Data, Hybrid Procedure.

1 Introduction

Many of today's economic systems are characterized by a network structure and operate in a dynamic environment [2]. Analytic methods seek to respect this development and adapt to these characteristics.

The Analytic Hierarchy Process (AHP) is a very popular method for setting priorities in hierarchical systems [4]. Network systems contain both positive and negative feedbacks. A variety of feedback processes create complex system behavior. For the whole network seems to be very appropriate Analytic Network Process (ANP) approach [5]. The ANP makes possible to deal systematically with all kinds of dependence and feedback in the system. The paper presents a dynamic approach. Dynamic ANP as an extension of ANP can deal with time dependent priorities in network systems. Dynamic models use concepts of state variables, flows, and feedback processes. The models try to reflect changes in real or simulated time and take into account that the network model components are constantly evolving.

Explaining the dynamic nature of systems is a subject of interest in research [2, 3, 6]. Section 3 of the paper is devoted to modeling time-dependent pairwise comparisons. This approach is suitable for long-term predictions. Another approach is based on prediction by exponential alignment of compositional data (Section 4). Composite data is the same for analyzing relative data, such as priorities. Exponential alignment of compositional data is appropriate for short-term forecast changes of priority. The paper proposes a hybrid procedure that combines and enriches each other's procedures (Section 5). Section 6 provides conclusions.

2 Analytic Network Process

The Analytic Hierarchy Process (AHP) is the method for setting priorities [4]. A priority scale based on reference is the AHP way to standardize non-unique scales in order to combine multiple performance measures. The AHP derives ratio scale priorities by making paired comparisons of elements on a common hierarchy level by using a 1 to 9 scale of absolute numbers. The absolute number from the scale is an approximation to the ratio w_j / w_k and then is possible to derive values of w_j and w_k. The AHP method uses the general model for synthesis of the performance measures in the hierarchical structure.

The Analytic Network Process (ANP) is the method [5] that makes it possible to deal systematically with all kinds of dependence and feedback in the performance system. The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements that share a set of attributes. At least one element in each of these clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements. Computations of the weights use three types of matrices.

Supermatrix

For the evaluation of all linkages, a pair-wise comparison method is used as for the AHP method. Pair-wise comparisons are inputs for calculating global priorities in the network system. The so-called supermatrix is a matrix that compares all the elements of the system to each other. Weights, calculated on the basis of pair-wise comparisons of the system elements, are the contents of individual supermatrix columns. Supermatrix is composed of sub-matrices comparing elements of one cluster with elements of another cluster W_{ij}. These matrices, if they are non-zero (they capture the effect of the elements of one cluster on elements of another cluster), are column stochastic, i.e. the sum of the elements in the column is equal to one. The sum of the elements in the supermatrix column is equal to the number of clusters being compared.

Weighted supermatrix

By pair-wise comparisons of each cluster gradually towards all clusters, we get the vectors of cluster weights. By multiplying the individual matrices W_{ij} of the supermatrix by the corresponding weights v_{ij}, we get from the supermatrix the so-called weighted supermatrix, capturing the importance of the clusters. Weighted supermatrix is already column stochastic and its elements express the assessment of the direct influence between the elements.

Limited supermatrix

If we create powers of the weighted supermatrix, these powers will express other indirect influences, given by links over other elements. After a certain number of iterations, the powers of weighted supermatrix are stabilized to the so-called limited matrix. The matrix columns are identical and represent the global priority of the elements.

We used the ANP software Super Decisions developed by Creative Decisions Foundation (CDF) for some experiments for testing the possibilities of the expression and evaluation of the network models (see Fig.1).

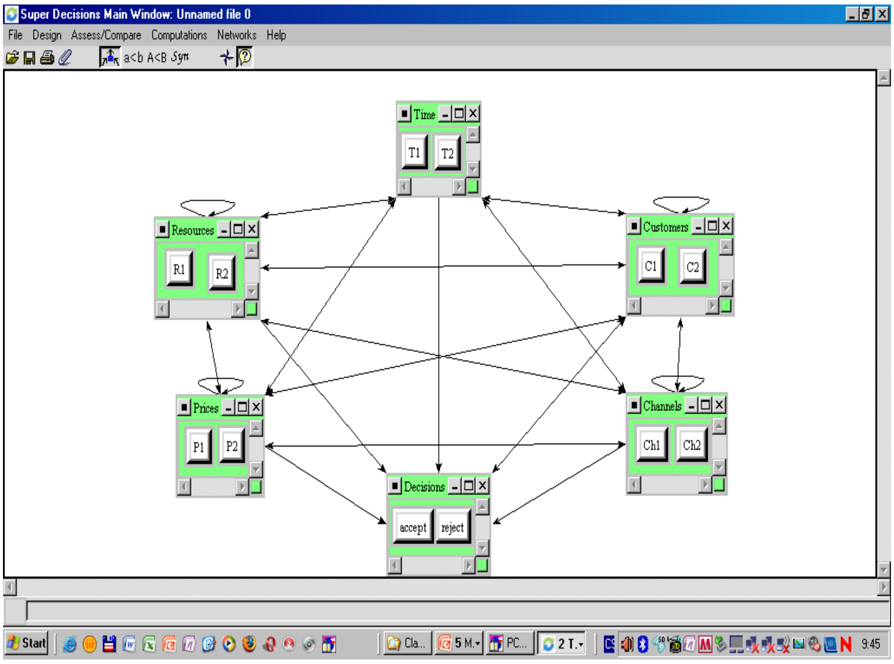

Fig. 1. Super Decisions

3 Time dependent priorities

Dynamic extensions of ANP method can work with time-dependent priorities in a networked system. There are two approaches for time-dependent pairwise comparisons:

- structural, by including scenarios,
- functional by explicitly involving time in the judgment process.

Functional dynamics is provided by pairwise comparison functions, where evaluations are time dependent. It is a generalization of ANP from points to functions.

For the functional dynamics, there are analytic or numerical solutions. The basic idea with the numerical approach is to obtain the time dependent priorities by simulation [6].

Judgment matrix in dynamic form:

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) \dots & a_{1k}(t) \\ a_{21}(t) & a_{22}(t) \dots & a_{2k}(t) \\ \vdots & \vdots & \vdots \\ a_{k1}(t) & a_{k2}(t) \dots & a_{kk}(t) \end{bmatrix} \tag{1}$$

By changes of time periods we get new weighs of elements. The ANP software Super Decisions can be used for computations of time dependent weights in discrete time periods.

Time dependent priorities capture long run trends but forecasting using pairwise comparison functions brings a problem with keeping the consistency of paired comparisons.

Example

We use the method for an illustration of positive feedback. The time dependent comparison of two products is expressed by S-curve:

$$a_{12}(t) = \frac{9}{1 + 7 \cdot 0{,}01^t} \tag{2}$$

The paired comparison matrix:

$$\begin{bmatrix} 1 & a_{12}(t) \\ 1/a_{12}(t) & 1 \end{bmatrix} \tag{3}$$

The numerical data are shown in Table 1 and plotted in Fig. 2.

Table 1. Dynamic comparisons.

t	$a_{12}(t)$	$w_1(t)$	$w_2(t)$
0	1,13	0,53	0,47
0.1	1,66	0,62	0,38
0.2	2,38	0,7	0,3
0.3	3,26	0,77	0,23
0.4	4,27	0,81	0,19
0.5	5,29	0,84	0,16
0.6	6,24	0,86	0,14
0.7	7,04	0,87	0,13
0.8	7,65	0,88	0,12

0.9	8,10	0,89	0,11
1	8,41	0,9	0,1

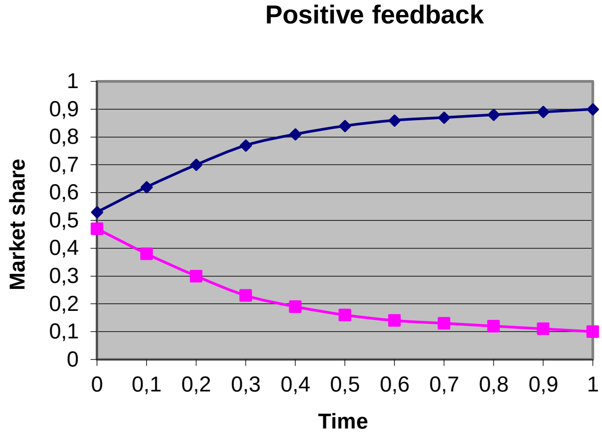

Fig. 2. Positive feedback – example.

4 Compositional data analysis

The compositional data are everywhere, where we need to work with data containing only relative information, which is useful for working with weights. A procedure based on exponential smoothing was designed, which is suitable for short-term predictions [3].

The following operations are defined on the simplex space:

$$S^k = \{\boldsymbol{x} = (x_1, x_2, \dots, x_k), x_i > 0, i = 1,2, \dots, k, \sum_{i=1}^{k} x_i = 1\} \quad (4)$$

Closure operator $\boldsymbol{C}(\mathbf{x})$: For any vector $\mathbf{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_k}) \in \boldsymbol{R}_+^k$

$$\boldsymbol{C}(\boldsymbol{x}) = \left(\frac{x_1}{\sum_{i=1}^{k} x_i}, \frac{x_2}{\sum_{i=1}^{k} x_i}, \dots, \frac{x_k}{\sum_{i=1}^{k} x_i}\right) \quad (5)$$

Perturbation: For any two vectors from simplex space **x**, **y** $\in S^k$

$$\boldsymbol{x} \oplus \boldsymbol{y} = \boldsymbol{C}(\boldsymbol{x_1 y_1}, \boldsymbol{x_2 y_2}, \dots, \boldsymbol{x_k y_k}) \quad (6)$$

Closer operator is used for Hadamard product of vectors **x** and **y**.

Power transformation: For any vector from simplex space **x** $\in S^k$ and $\alpha \in R_+$

$$\boldsymbol{\alpha} \otimes \boldsymbol{x} = \boldsymbol{C}(\boldsymbol{x}_1^{\boldsymbol{\alpha}}, \boldsymbol{x}_2^{\boldsymbol{\alpha}}, \dots, \boldsymbol{x}_k^{\boldsymbol{\alpha}}) \tag{7}$$

Difference:

$$\boldsymbol{x} \ominus \boldsymbol{y} = \boldsymbol{x} \oplus (-\mathbf{1} \otimes \boldsymbol{y}) \tag{8}$$

Exponential smoothing with compositional data can be used for predicting weights

$$\boldsymbol{w}_t = (w_{t1}, w_{t2}, \dots, w_{tk}), w_{ti} > 0, i = 1,2, \dots, k, \sum_{i=1}^{k} w_{ti} = 1 \tag{9}$$

in a short time.

Simple exponential smoothing
Vector of observations at time *t*

$$\boldsymbol{x}_t = (x_{t1}, x_{t2}, \dots, x_{tk}), x_{ti} > 0, i = 1,2, \dots, k, \sum_{i=1}^{k} x_{ti} = 1 \tag{10}$$

elements of simplex space.

Vector of predictions at time *t*

$$\boldsymbol{y}_t = (y_{t1}, y_{t2}, \dots, y_{tk}), y_{ti} > 0, i = 1,2, \dots, k, \sum_{i=1}^{k} y_{ti} = 1 \tag{11}$$

elements of simplex space.

The formula for simple exponential smoothing of compositional data:

$$\boldsymbol{y}_t = \alpha \otimes \boldsymbol{x}_{t-1} \oplus (1 - \alpha) \otimes \boldsymbol{y}_{t-1} \tag{12}$$

Double exponential smoothing
We introduce for trend modeling a vector of trend values $\mathbf{u}_t$, a vector of slopes $\mathbf{v}_t$, a smoothing constant $0 \le \alpha \le 1$, a trend constant $0 \le \beta \le 1$.

Formulas for double exponential smoothing of compositional data:

$$\boldsymbol{u}_t = \alpha \otimes \boldsymbol{x}_t \oplus (1 - \alpha) \otimes (\boldsymbol{u}_{t-1} \oplus \boldsymbol{v}_{t-1}) \tag{13}$$

$$\boldsymbol{v}_t = \beta \otimes (\boldsymbol{u}_t \ominus \boldsymbol{u}_{t-1}) \oplus (1 - \beta) \otimes \boldsymbol{v}_{t-1} \tag{14}$$

$$\boldsymbol{y}_t = \boldsymbol{u}_{t-1} \oplus \boldsymbol{v}_{t-1} \tag{15}$$

5 Hybrid procedure

For a dynamic version of the ANP method, we propose a hybrid procedure that combines the advantages of long-term prediction of pair-wise comparisons and short-term predictions by exponential smoothing of compositional data. This procedure also enriches each of these processes by obtaining more accurate data. Both procedures were presented in the previous sections and here we limit ourselves to a brief summary of the hybrid procedure steps:

- **Step 1:** Formulation of pair-wise comparison functions.
- **Step 2:** Testing and improving consistency of comparisons.
- **Step 3:** Collection of historical data by ANP priorities over time.
- **Step 4:** Using of compositional exponential smoothing.
- **Step 5:** Selection of the best coefficient α, β with lowest value of error.
- **Step 6:** Forecasting of priorities for next time periods.
- **Step 7:** Re-formulation of comparison functions based on short-run model, go to step 2.

6 Conclusions

The proposed hybrid procedure is an attempt to eliminate the shortcomings of both procedures and enrich their advantages and benefits. Searching for a tool for evaluating dynamic network models is an important research area. There are some possibilities to modify and to generalize the approach. Using such a tool in practice would have numerous applications.

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