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Sortino's Ratio for Oriented Fuzzy Discount Factor

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Abstract: Financial equilibrium criteria are very important tool for generating investment strategy. Obtained in this manner investment strategies base on analysis of distinguished profit index. In the article, investment strategies use a comparison of a profit index and related value limit. In considered formal model, the imprecise present value is evaluated by means of a trapezoidal oriented fuzzy number (Tr-OFN). Then expected discount factor is evaluated by Tr-OFN too. Its imprecise value can be used as a premise for financial decision making. For this reason, the Sortino's Ratio criterion is generalized for the case of expected discount factor described by Tr-OFN. Then proposed investment strategies use a comparison of oriented fuzzy profit index and related crisp value limit. In this manner an investor can obtain imprecise investment recommendation described by a fuzzy subset in rating scale. Results obtained show that generalized Sortino's Ratio may be applied in support systems for investment making. All theoretical results are illustrated by simple examples.

Keywords: Sortino's ratio criterion; oriented fuzzy number; fuzzy oriented discount factor

JEL Classification: C44; C02; G10

1. Introduction

In general, present value (PV) is determined as current equivalent of evaluated cash flow (Piasecki 2012). It is widely accepted that PV of future cash flow can be evaluated by fuzzy number (FN). Then an expected return rate is described by fuzzy subset in the family of all real numbers. This fact is a theoretical base for investment strategies presented by Piasecki (2014). Moreover, Piasecki and Siwek (2018) show that the fuzzy expected discount factor is a better tool for securities management than the fuzzy expected return rate. For this reason, an expected discount factor is applied here as premise for investment making.

Ordered FN is defined by Kosiński et al (1993; 2002; 2003). For formal reason, Piasecki (2018) revise the original Kosiński's theory. Let us note that if any ordered FN is determined with use the revised theory then it is called Oriented FN (OFN).

The aim of this paper is extension of introduced by Piasecki (2014) investment strategies for the case when PV is evaluated by OFN. Then PV is additionally equipped with forecast of PV's changes. The first attempt to this subject was presented in (Łyczkowska-Hanćkowiak and Piasecki 2019). Here, we use our experience gathered during our work on the other criteria. Therefore, here it is presented a revised approach to considered extension. All obtained results are used for extension the Sortino's Ratio criterion (Sortino and Price 1994)) to the case of PV described by OFN.

This paper is organized in following way. Section 2 describes OFNs and their chosen properties. In Section 3 PV is presented with use trapezoidal OFNs. In Section 4, the formula for oriented fuzzy expected discount factor is derived. An upgraded model for investment recommendations is presented in Section 5. The extended Sortino's Ratio Criterion is given in Section 6. Final conclusions and proposed future research directions are given in Section 7

2. Oriented Fuzzy Numbers – Basic Facts

Considered objects can be modelled as elements of given space $\mathbb{X}$. The widely accepted tool for any imprecise classification of these elements is fuzzy subset (Zadeh 1967). Each fuzzy subset $\mathcal{A}$ is unambiguously distinguished by its membership function $\mu_A \in [0,1]^{\mathbb{X}}$. From the point-view of multi-valued logic (Łukasiewicz 1922/23), the value $\mu_A(x)$ may be interpreted as a truth value of the sentence "$x \in \mathcal{A}$". The symbol $\mathcal{F}(\mathbb{X})$ denotes the family of all fuzzy subsets of the space $\mathbb{X}$.

Dubois and Prade (1978) have introduced fuzzy number (FN) as such fuzzy subset in the space $\mathbb{R}$ which can be considered as an imprecise estimation of real number. The ordered FNs are defined as an FN extension (Kosiński et al. 1993, 2002, 2003). Ordered FNs helpfulness is the result of interpretability them as FNs additionally equipped with information about the location of estimated number. Currently, ordered FNs defined by Kosiński are frequently called the Kosiński's numbers (Prokopowicz and Pedrycz 2015, Prokopowicz 2015, Piasecki 2019). A discussion on the present state of knowledge on Kosiński's numbers is presented in (Prokopowicz et al. 2015). A major disadvantage of Kosiński's theory is existence such Kosiński's numbers which cannot be represented by membership function. (Kosiński 2006). Therefore, this theory is revised by Piasecki (2018a). If an ordered FN is determined with use of the revised definition, then it is called Oriented FN (OFN).

In this article, all considerations are restricted to the case of Trapezoidal OFN (TrOFN) defined as follows.

Definition 1. (Piasecki 2018a) For any monotonic sequence $(\mathrm{a,b,c,d}) \subset \mathbb{R}$, the TrOFN $\overleftrightarrow{Tr}(a,b,c,d) = \overleftrightarrow{\mathcal{T}}$ is determined as the pair of the orientation $\langle a \rightarrowtail b\rangle = (a,d)$ and the fuzzy subset $\mathcal{T} \in \mathcal{F}(\mathbb{R})$ distinguished by its membership functions $\mu_T \in [0,1]^{\mathbb{R}}$ given as follows

$$\mu_T(x) = \mu_{Tr}(x|a,b,c,d) = \begin{cases} 0, & x \notin [a,d] \equiv [d,a], \\ \frac{x-a}{b-a}, & x \in [a,b[\equiv]b,a], \\ 1, & x \in [b,c] \equiv [c,b], \\ \frac{x-d}{c-d}, & x \in]c,d] \equiv [d,c[. \end{cases} \tag{1}$$

By symbol $\mathbb{K}_{Tr}$ we denote the space of all TrOFNs. For $a < d$, $\overleftrightarrow{Tr}(a,b,c,d)$ is positively oriented. Then $\overleftrightarrow{Tr}(a,b,c,d)$ is an image of term "about or slightly above z" expressed for any $z \in [b,c]$. For $a > d$, $\overleftrightarrow{Tr}(a,b,c,d)$ is negatively oriented. Then $\overleftrightarrow{Tr}(a,b,c,d)$ is an image of term "about or slightly below z" expressed for any $z \in [b,c]$. For $a = d$, $\overleftrightarrow{Tr}(a,a,a,a) = [\![a]\!]$ describes un-oriented real number $a \in \mathbb{R}$.

On the space $\mathbb{K}_{Tr}$ the relation $\overleftrightarrow{\mathcal{K}}.\widetilde{GE}.\mathcal{L}$ is defined in following way

$$\overleftrightarrow{\mathcal{K}}.\widetilde{GE}.\overleftrightarrow{\mathcal{L}} \Leftrightarrow \text{"TrOFN } \overleftrightarrow{\mathcal{K}} \text{ is greater or equal to TrOFN } \overleftrightarrow{\mathcal{L}}.\text{"} \tag{2}$$

Above relation is a fuzzy preorder $\widetilde{GE} \in \mathcal{F}(\mathbb{K}_{Tr} \times \mathbb{K}_{Tr})$ described by its membership function $\nu_{GE} \in [0,1]^{\mathbb{K}_{Tr}\times\mathbb{K}_{Tr}}$ firstly considered by Piasecki (2018a; 2019). Due these results, for any pair $(\overleftrightarrow{Tr}(a,b,c,d), h) \in \mathbb{K}_{Tr} \times \mathbb{R}$ we have:

$$\nu_{GE}\left(\overleftrightarrow{Tr}(a,b,c,d), [\![h]\!]\right) = \begin{cases} 0, & h > \max\{a,d\}, \\ \frac{h-\max\{a,d\}}{\max\{b,c\}-\max\{a,d\}}, & \max\{a,d\} \geq h > \max\{b,c\}, \\ 1, & \max\{b,c\} \geq h, \end{cases} \tag{3}$$

$$\nu_{GE}\left([\![h]\!], \overleftrightarrow{Tr}(a,b,c,d)\right) = \begin{cases} 0, & h < \min\{a,d\}, \\ \frac{h-\min\{a,d\}}{\min\{b,c\}-\min\{a,d\}} & \min\{a,d\} \leq h < \min\{b,c\}, \\ 1 & \min\{b,c\} \leq h. \end{cases} \tag{4}$$

3. Oriented Fuzzy Present Value

Any PV can be imprecise. It implies that PV must be evaluated by FN. Kuchta (2000) justifies the using trapezoidal FNs for PV evaluating. Moreover, PV estimation should be equipped with prediction of next PV changes. For these reasons, PV is estimated by TrOFN.

$$\overleftrightarrow{PV} = \overleftrightarrow{Tr}(V_s, V_f, V_l, V_e), \tag{5}$$

where the monotonic sequence $(V_s, V_f, \check{C}, V_l, V_e)$ is defined as follows

- $\check{C}$ – market price,
- $[V_s, V_e] \subset \mathbb{R}^+$ is given interval of all possible PV values,
- $[V_f, V_l] \subset [V_s, V_e]$ is given interval of all prices not significantly different from the market price $\check{C}$.

If $V_s < V_e$, then the positive PV orientation is a prediction of the PV increase. For $V_s > V_e$, the negative PV orientation is the forecast of the decrease in PV. Such PV is called the oriented PV (OPV).

Example 1: (Łyczkowska-Hanćkowiak 2019): We consider the financial portfolio π containing the shares in following stock companies: Assecopol (ACP), ENERGA (ENG), JSW (JSW), KGHM (KGH), LOTOS (LTS), ORANGEPL (OPL) and PKOBP (PKO). All above stock companies are quoted on the Warsaw Stock exchange (WSE). Based on session closing on WSE on January 15, 2018, for each evaluated share we determine its OPV as TrOFN representing its Japanese candle (Nison 1991). Determined in this manner shares' OPVs are shown in Table 1 (Łyczkowska and Piasecki 2018a). For each portfolio component, we determine its market price $\check{C}_s$ as initial price on 16.01.2018.

Table 1. Evaluation of stocks from portfolio π.

Stock Company	OPV $\overleftrightarrow{PV}_s$	Market Price $\check{C}_s$	Expected Return Rate $\bar{r}_s$	Downside Semi Variance ς_s^2
ACP	$\overleftrightarrow{Tr}(45.90;\ 45.90;\ 45.50;\ 45.48)$	45.70	0.0300	0.000050
CPS	$\overleftrightarrow{Tr}(22.92;\ 22.82;\ 22.82;\ 22.76)$	22.82	0.0355	0.000100
ENG	$\overleftrightarrow{Tr}(10.22;\ 10.19;\ 10.17;10.14)$	10.18	0.0150	0.000015
JSW	$\overleftrightarrow{Tr}(92.24;\ 92.54;92.54;92.80)$	92.54	0.0400	0.000150
KGH	$\overleftrightarrow{Tr}(102.65;\ 103.05;103.60;\ 103.90)$	103.33	0.0390	0.000105
LTS	$\overleftrightarrow{Tr}(56.70;\ 56.56;\ 56.40;\ 56.28)$	56.48	0.0450	0.000210
OPL	$\overleftrightarrow{Tr}(5.75;\ 5.76;\ 5.90;\ 5.90)$	5.83	0.0360	0.000160
PGE	$\overleftrightarrow{Tr}(10.39;\ 10.39;10.35;\ 10.33)$	10.37	0.0235	0.000100
PKO	$\overleftrightarrow{Tr}(42.61;\ 42.61;\ 43.22;\ 43.22)$	42.91	0.0420	0.000205

We notice that the stock companies JSW, KGH, OPL and PKO are evaluated by OPV having positive orientation. Then OPV predicts a rise in market price. Similarly, the stock companies ACP, CPS, ENG, LTS and PGE are evaluated by OPV with negative orientation. In this case, OPV predicts a fall in market price.

4. Oriented Fuzzy Discount Factor

We use the simple return rate as basic characteristic of benefits from owning considered security. Let the uncertainty risk be described by probability distribution of return rate. If expected value of this distribution exists, then it is equal to expected return rate $\bar{r}$. Then expected discount factor (EDF) $\bar{v} \in \mathbb{R}$ is defined as follows:

$$\bar{v} = \frac{1}{1+\bar{r}} \ . \qquad (6)$$

It is obvious that the maximum criterion formulated for an expected return rate may be equivalently replaced by the minimum criterion formulated for EDF.

Example 2: In all examples, we consider quarterly duration of investment. For each component of portfolio π, we calculate its return rate and related downside semi variance. All results of these calculations are shown in Table 1.

In (Łyczkowska-Hanćkowiak and Piasecki 2018b) it is proved that if oriented EDF (OEDF) is determined by OPV (5) then it is described by TrOFN

$$\overleftrightarrow{V} = \overleftrightarrow{Tr}\left(\frac{V_s}{\check{C}} \cdot \bar{v}, \frac{V_f}{\check{C}} \cdot \bar{v},\ \frac{V_l}{\check{C}} \cdot \bar{v}, \frac{V_e}{\check{C}} \cdot \bar{v}\right). \qquad (7)$$

Example 3: Using (6) and (7), we calculate EDF and OEDF for each share belonging to considered portfolio π. Obtained in this manner evaluations are shown in Table 2.

The discount factor calculated in this manner is TrOFN with the identical orientation as OPV used for estimation.

5. Investment Recommendations

We consider a recommendation given by an advisor to an investor. Any recommendation is a subset of rating scale. In this paper, all recommendations are formulated with use rating scale applied in (Piasecki 2014). Used rating scale is described by the set $\mathbb{A} = \{A^{++}, A^{+}, A^{0}, A^{-}, A^{--}\}$, where

- A^{++} is the advice "Buy";
- A^{+} is the advice "Accumulate";
- A^{0} is the advice "Hold";
- A^{-} is the advice "Reduce";
- A^{--} is the advice "Sell".

Let fixed security $\check{S}$ be represented by the pair $(\bar{r}_s, \varpi_s)$ of the expected return $\bar{r}_s$ and the parameter ϖ_s characterizing the uncertainty risk related to investing in represented security $\check{S}$. The symbol $\mathbb{S}$ denotes the portfolio containing all considered securities. Any recommendation depends on the mentioned above pair of parameters. The criterion for advices choice may be presented as a comparison between the profit values $g(\bar{r}_s|\varpi_s)$ and the profitability threshold (PT) $\check{G}$. Introduced above function $g(\cdot\,|\varpi_s): \mathbb{R} \to \mathbb{R}$ increases with the expected return rate. Then any recommendation is formulated with use the choice function $\Lambda: \mathbb{S} \times \mathbb{R} \to 2^{\mathbb{A}}$ was given in following way (Piasecki 2014)

- $A^{++} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow g(\bar{r}_s|\varpi_s) > \check{G} \Leftrightarrow \neg g(\bar{r}_s|\varpi_s) \leq \check{G},$ (8)
- $A^{+} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow g(\bar{r}_s|\varpi_s) \geq \check{G},$ (9)
- $A^{0} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow g(\bar{r}_s|\varpi_s) = \check{G} \Leftrightarrow \wedge\, g(\bar{r}_s|\varpi_s) \geq \check{G} \wedge g(\bar{r}_s|\varpi_s) \leq \check{G},$ (10)
- $A^{-} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow g(\bar{r}_s|\varpi_s) \leq \check{G},$ (11)
- $A^{--} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow g(\bar{r}_s|\varpi_s) < \check{G} \Leftrightarrow \neg g(\bar{r}_s|\varpi_s) \geq \check{G}.$ (12)

This way was assigned the subset $\Lambda(\check{S}, \check{G}) \subset \mathbb{A}$ describing the recommendation granted the security$\check{S}$.

The security $\check{S}$ may be equivalently represented by the ordered pair $(\bar{v}_s, \varpi_s)$, where $\bar{v}_s$ is EDF determined by (6). Then we have

$$g(\bar{r}_s|\varpi_s) \geq \check{G} \Leftrightarrow \bar{v}_s \leq \frac{1}{1+g^{-1}(\check{G}|\varpi_s)} = H_s(\check{G}), \quad (13)$$

$$g(\bar{r}_s|\varpi_s) \leq \check{G} \Leftrightarrow \bar{v}_s \geq H_s(\check{G}). \quad (14)$$

The value $H_s(\check{G})$ may be applied in any comparison with EDF as such profitability threshold (SPT) which is specified for each security $\check{S}$ separately. Then the choice function $\Lambda: \mathbb{S} \times \mathbb{R} \to 2^{\mathbb{A}}$ is equivalently determined as follows

- $A^{++} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow \neg \bar{v}_s \geq H_s(\check{G}),$ (15)
- $A^{+} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow \bar{v}_s \leq H_s(\check{G}),$ (16)
- $A^{0} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow \bar{v}_s \leq H_s(\check{G}) \wedge \bar{v}_s \geq H_s(\check{G}),$ (17)
- $A^{-} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow \bar{v}_s \geq H_s(\check{G}),$ (18)
- $A^{--} \in \Lambda(\check{S}, \check{G}) \Leftrightarrow \neg \bar{v}_s \leq H_s(\check{G}).$ (19)

Let the security $\check{S}$ be represented by such ordered pair $(\vec{V}_s, \varpi_s)$ that $\vec{V}_s \in \mathbb{K}_{Tr}$ is OEDF calculated with use (7). Then any value of choice function $\tilde{\Lambda}(\check{S}, \check{G})$ is described by fuzzy subset in rating scale $\mathbb{A}$. This fuzzy subset is determined by its membership function $\lambda(\cdot\,|\check{S}, \check{G}): \mathbb{A} \to [0,1]$ defined in line with (15) – (19) as follows:

- $\lambda(A^{++}|\check{S}, \check{G}) = 1 - \nu_{GE}(\vec{V}_s, [\![H_s(\check{G})]\!]),$ (20)
- $\lambda(A^{+}|\check{S}, \check{G}) = \nu_{GE}([\![H_s(\check{G})]\!], \vec{V}_s),$ (21)

- $\lambda(A^0|\check{S},\check{G}) = \min\{\nu_{GE}([\![H_s(\check{G})]\!],\overleftrightarrow{V_s}),\ \nu_{GE}(\overleftrightarrow{V_s},[\![H_s(\check{G})]\!])\}$, (22)
- $\lambda(A^-|\check{S},\check{G}) = \nu_{GE}(\overleftrightarrow{V_s},[\![H_s(\check{G})]\!])$, (23)
- $\lambda(A^{--}|\check{S},\check{G}) = 1 - \nu_{GE}([\![H_s(\check{G})]\!],\overleftrightarrow{V_s})$. (24)

From the point-view of investment-making, the value $\lambda(A|\check{S},\check{G})$ is interpreted as a degree of advice support $A \in \mathbb{A}$, i.e. a declared adviser's participation in responsibility for the final investment decision in accordance with the recommendation $\{A\} \subset \mathbb{A}$.

6. The Sortino's Ratio

The Sortino's Ratio (Sortino and Price 1997) is a tool for risk management under financial equilibrium. In any financial equilibrium criterion, this model, we compare the expected return rate $\bar{r}_s$ from considered security and the expected return rate $\bar{r}_M$ from the distinguished market portfolio. We consider the advice choice function where profit index and limit value are determined by Sortino's Ratio. Then profit index evaluates amount of specific unit premium for loss risk. Moreover, the limit value evaluates amount of the market unit premium for loss risk. The benchmarks of our assessment is a market portfolio represented by such ordered pair $(\bar{r}_M, \varsigma_M{}^2)$, where the downside semi variance $\varsigma_M{}^2$ evaluates the market loss risk. The reference point is a risk-free bond instrument represented by the ordered pair $(r_0, 0)$, where r_0 is a free of risk return rate.

Example 4: We focus on the WSE. We consider risk-free financial instrument determined as quarterly treasure bound with return rate $r_0 = 0.0075$. The market portfolio is defined as portfolio designating stock exchange index WIG20. The market portfolio is represented by the ordered pair $(r_M, \varsigma_M{}^2) = (0.0200, 0.000015)$.

Considered security $\check{S}$ is represented by the ordered pair $(\bar{r}_s, \varsigma_S^2)$, where downside semi variance $\varsigma_S{}^2$ evaluates a loss risk. Then Sortino and Price (1997) define the profit index $g(\cdot\,|\varsigma_S)\colon \mathbb{R} \to \mathbb{R}$ and the limit value PT $\check{G}$ as follows:

$$g(\bar{r}_s|\varsigma_S) = \frac{r_S - r_0}{\varsigma_S}, \tag{25}$$

$$\check{G} = \frac{r_M - r_0}{\varsigma_M}. \tag{26}$$

For this case, we calculate SPT $H_s(\check{G})$ as follows

$$H_s(\check{G}) = \frac{\varsigma_M}{\varsigma_S \cdot (r_M - r_0) + \varsigma_M \cdot (r_0 + 1)}. \tag{27}$$

Example 5: Using (27), we calculate SPT for each security belonging to the portfolio π. Evaluations obtained in this way are presented in Table 2.

Table 2. EDFs and OEDFs of securities belonging to the portfolio π.

Stock Company	**EDF $\bar{v}_s$**	**OEDF $\overleftrightarrow{V}_s$**	**SPT H_s**
ACP	0.9709	$\overleftrightarrow{Tr}(0.9751; 0.9751; 0.9666; 0.9662)$	0.9706
CPS	0.9657	$\overleftrightarrow{Tr}(0.9699; 0.9657; 0.9657; 0.9632)$	0.9618
ENG	0.9852	$\overleftrightarrow{Tr}(0.9891; 0.9862; 0.9842; 0.9813)$	0.9804
JSW	0.9615	$\overleftrightarrow{Tr}(0.9584; 0.9615; 0.9615; 0.9642)$	0.9551
KGH	0.9625	$\overleftrightarrow{Tr}(0.9592; 0.9599; 0.9650; 0.9678)$	0.9610
LTS	0.9569	$\overleftrightarrow{Tr}(0.9606; 0.9583; 0.9555; 0.9535)$	0.9485
OPL	0.9652	$\overleftrightarrow{Tr}(0.9520; 0.9536; 0.9768; 0.9768)$	0.9539
PGE	0.9770	$\overleftrightarrow{Tr}(0.9789; 0.9789; 0.9751; 0.9732)$	0.9618
PKO	0.9597	$\overleftrightarrow{Tr}(0.9530; 0.9530; 0.9666; 0.9666)$	0.9490

For each considered security, by means of (20) – (24) we calculate membership functions of investment recommendations presented in Table 3.

Table 3. Membership functions of recommendations.

	Investment Recommendation				
Stock Company	A^{--}	A^{-}	A^{0}	A^{+}	A^{++}
ACP	0	1	1	1	0
CPS	1	1	0	0	0
ENG	1	1	0	0	0
JSW	1	1	0	0	0
KGH	0	1	1	1	0
LTS	1	1	0	0	0
OPL	0	1	1	1	0
PGE	1	1	0	0	0
PKO	1	1	0	0	0

We see that obtained recommendations are ambiguous. These recommendations are only the opinion of the adviser. The final investment decision should be made by investor.

7. Conclusions

Presented results can be used in behavioural finance quantitative theory of behavioural finance as a part of model of investors' decisions. These results can also form theoretical basis for construction of investment decision-making support system.

For these portfolio, Sharpe's Ratio gave recommendations (Łyczkowska-Hanćkowiak 2019) which are different from the recommendations obtained by means of Sortino's Ratio. This fact results from the difference between the economic nature of both ratios. Sharpe's Ratio assesses the unit premium for risk, while Sortino's Ratio assesses the unit premium for loss risk.

Presented results can be a well starting point for future investigation of the impact of oriented imprecision on risk burdening investment decision making.

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