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Equilibrium Searching in Supply Chains by Biform Games

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Abstract: Supply chain management is a discipline that has met with great interest in both practical applications and theoretical development. The supply chain consists of independent units that cooperate and compete in different situations. Finding balance in supply chains is a difficult task. The paper proposes a model framework that captures the existence of multiple units with different interests and preferences, which are evaluated by multiple evaluation criteria. The procedure is based on biform games that incorporate cooperative and non-cooperative procedures. The authors' contribution is the division of biform games into sequential and simultaneous forms. Sequential biform games gradually apply cooperative and subsequently non-cooperative techniques. Simultaneous shape contemplates the simultaneous use of cooperative and non-cooperative techniques. The search for equilibrium is based on negotiating the aspiration values of the evaluation criteria. A supply chain equilibrium is when non-empty intersection of these values is achieved.

Keywords: supply chain; biform game; equilibrium

JEL Classification: C70

1. Introduction

Game theory is a discipline that analyzes situations with conflicting interests of the participants. Such problems often arise and affect the behavior of participants in economic situations. The classic work of John von Neumann and Oskar Morgenstern (1944) has already formulated basic models of game theory for economic decision-making. Game theory has developed considerably and a considerable amount of literature has been published. Kreps (1991) and Myerson (1997) provide an overview of basic models, concepts, and practices in game theory. Models of game theory analyze conflict situations in which players have their interests and it is necessary to seek a balanced solution. Classical game theory is divided into cooperative non-cooperative concepts. Nash equilibrium is a classic concept in non-cooperative theory, when this situation means that when any player withdraws from his equilibrium position while maintaining the positions of other players, he cannot improve his winnings. The cooperative game theory analyzes the possible common winnings of the players, the conditions under which they are formed, how the coalitions of players are formed, and how they redistribute the winnings. Also included is an analysis of the stability of coalitions of players and their winnings. Brandenburger and Stuart (2007) suggest biform games as a connection of non-cooperative and cooperative games.

Supply chain management is a discipline that has met with great interest in both practical applications and theoretical development (Tayur et al. 2012). The supply chain consists of independent units that cooperate and compete in different situations. In recent years, game theory has provided a number of models and techniques for supply chain management analysis. Cachon and Netessine (2004) provide an excellent overview of the concepts and practices of non-cooperative game theory for supply chain analysis. This concept forms allocation mechanisms analogously to the classical market environment.

Nagarajan and Sošić (2008), on the other hand, provide an overview of existing literature on the use of cooperative game models and practices in supply chain management. The authors focused mainly on the achievable common outputs, their redistribution, formation and stability of coalitions.

Brandenburger and Nalebuff (2011) introduce the concept of co-opetition, which captures the fact that collaboration is applicable in some cases, while competition is more appropriate in others. The authors propose biform games to explain and justify the proposed concept. Okura and Carfi (2014) analyze how cooperative studies can be linked to game theory models and procedures.

The paper proposes a modeling framework for equilibrium searching in supply chains based on biform games. The authors' contribution is the division of biform games into sequential and simultaneous forms. Sequential shape gradually applies cooperative and subsequently non-cooperative techniques. Simultaneous shape contemplates the simultaneous use of cooperative and non-cooperative techniques. The search for equilibrium is based on negotiating the aspiration values of the evaluation criteria. A supply chain equilibrium is found when non-empty intersection of these values is achieved.

The rest of the paper is arranged as follows. The elements of supply chains are summarized in Section 2. Section 3 reports on sequential biform games in supply chain analyses. The practices of simultaneous biform games are described in Section 4. The discussion and the conclusions are presented in Section 5.

2. Supply Chain

A supply chain is defined as a dynamic complex network structure of units, resources, activities, information and technologies linked to meet demand and move a product from the initial supplier to the final consumer (Fiala 2005).

The supply chain is modeled as a network system with clusters of:

- suppliers,
- manufacturers,
- distributors,
- retailers,
- customers,

where

- material,
- information,
- financial,
- and decision

flows connect units in the supply chain. The flows progress in both directions. Decision flows are considered as a sequence of decisions between supply chain units.

Supply chain management is a collection of models, tools and techniques that are used to manage these systems throughout the life cycle. The supply chain management consists of four successive phases:

- design,
- control,
- performance evaluation,
- and performance improvement.

These phases repeatedly alternate during the dynamic development of the supply chain and the environment in which the chain is formed. Supply chain performance is evaluated by more evaluation criteria:

- economic,
- social,
- environmental,
- and others.

Models of game theory are very useful in analyzing and managing supply chains due to the inclusion of a larger number of decision-making units that have conflicting but also some common interests. The proposed biform game models prove to be a suitable tool for finding a equilibrium in a network system, where competitive and cooperative processes between the units of the system take place. The model is enriched by the introduction of multiple evaluation criteria to measure supply chain performance. The search for an equilibrium in supply chains is modeled using negotiation techniques under pressure. Negotiation brings the exchange of information, specifying material flows, reducing inefficiencies and, leads to a better functioning of supply chains according to evaluation criteria, and hence there is a performance improvement of supply chains.

3. Sequential Biform Game

The sequential biform game is composed from two stages. In the first stage, players compete and use instruments of non-cooperative games. In the second stage, players cooperate and the tools of cooperative games are used.

In the first stage, the concept of Nash equilibrium is applied. Nash equilibrium is a set of equilibrium strategies, from which no player can improve his payout that departs from his equilibrium strategy, and other players remain with their equilibrium strategies.

A non-cooperative game in the normal form is given by this formula

$$\{N = \{1,2, \dots, n\};\ X_1, X_2, \dots, X_n;\ \pi_1(x_1, x_2, \dots, x_n), \pi_2(x_1, x_2, \dots, x_n), \dots, \pi_n(x_1, x_2, \dots, x_n)\ \}, \quad (1)$$

where N is a set of n players; X_i, i = 1, 2, …, n, is a set of strategies for player i; $\pi_i(x_1, x_2, \dots, x_n)$, i = 1, 2, …, n, is a player's i payout function, defined on n sets X_i, i = 1, 2, …, n.

Strategies of all players than player i are defined by a vector

$$\mathbf{x}_{-i} = (x_1, \dots, x_{i-1}, x_{i+1} \dots, x_n). \quad (2)$$

A vector of strategies $(x_1^0, x_2^0, \dots, x_n^0)$ is Nash equilibrium if the following conditions are satisfied

$$x_i^0(\mathbf{x}_{-i}^0) = \underset{x_i}{\operatorname{argmax}}\, \pi_i(x_i, \mathbf{x}_{-i}), i = 1,2, \dots, n. \quad (3)$$

In second stage, a cooperative game approach is used to get the maximal common output and to distribute this output to individual players. Shapley values (8) can be used for distribution of this output total.

The maximal common output is reached if the next problem is solved

$$\mathbf{x}^0 = \underset{\mathbf{x}}{\operatorname{argmax}} \sum_{i=1}^{n} \pi_i(x_i). \quad (4)$$

The game in the characteristic function form is advantageous for modeling and solving cooperative games. The characteristic function $\upsilon(S)$ is introduced for all subsets $S \subseteq N$ (i.e. for all coalition) and defines values $\upsilon(S)$ by following formulas:

$$\upsilon(\emptyset) = 0, \upsilon(S_1 \cup S_2) \geq \upsilon(S_1) + \upsilon(S_2), \quad (5)$$

where S_1, S_2 are disjoint subsets of the set of all players N. A cooperative game with set N of all players in the characteristic function form is defined as the pair (N, υ).

Shapley (1953) introduced a specific allocation rule that has positive characteristics in terms of equilibrium and fairness of distribution. Shapley vector is defined as

$$\mathbf{h} = (h_1, h_2, \dots, h_n), \quad (6)$$

where the elements of the vector mean the average marginal contribution of the i-th player to all coalitions in which he may appear as a participant. A contribution of the player i to the coalition S is designed by the difference:

$$\upsilon(\mathrm{S}) - \upsilon(S - \{i\}). \quad (7)$$

Shapley value for the player i is designed as a weighted sum of marginal contributions by the formula:

$$h_i = \sum_S \left\{ \frac{(|S|-1)!(n-|S|)!}{n!} [\upsilon(S) - \upsilon(S - \{i\})] \right\}, \quad (8)$$

where the number of coalition participants is denoted by the symbol $|S|$ and summarizing takes place across all coalitions where $i \in S$.

It is useful to link these two stages together. Confidence indices $0 \leq \alpha^i \leq 1$, for all $i = 1, 2, \ldots, n$, are presented to create the connection between the non-cooperative and cooperative stages.

4. Simultaneous Biform Games

The simultaneous biform game is composed from one stage where a mix of approaches for cooperative and non-cooperative games is used together. Multi-round negotiations are in the progress in the one-stage model. The specific combination of these approaches varies depending on the situation of the problem. The problem needs to be analyzed, especially in terms of which players can cooperate and to what extent. There are two specific extreme cases. A classical cooperative model (4) can be used if all players can fully collaborate. The subsequent distribution of the output is based on the Shapley values (8). A classical non-cooperative model (3) can be used if no one can cooperate even in a partial extent.

The general simultaneous biform game model is based on multi-round negotiations with multiple evaluation criteria (Fiala 1999). The concept of negotiation under pressure goes out from the fact that each player is exposed to different internal and external pressures. The extent of cooperation is created by the set of constraints that arise dynamically according to pressures. The effects of pressures are transformed into the constraints of the model.

4.1. Negotiation model

The general negotiation model supposes n players. A strategy space for the negotiation process is denoted as X. Strategies are vectors $\mathbf{x} \in X$, whose components express the parameter values of the strategy. A consensus strategy $\mathbf{x}^*$ is an element of the strategy space X. The classical game concepts are based on a fixed structure of the game and sets of strategies are fixed also. In the proposed model, sets of strategies and evaluations of strategies are considered as dynamic $X_i(t)$, $i = 1, 2, \ldots, n$. Changes take place in the discrete time points $t = 0, 1, 2, \ldots, T$.

Each player evaluates strategies by multiple evaluation criteria and assesses the strategies according to the target values. We denote $\mathbf{f}^1(\mathbf{x})$, $\mathbf{f}^2(\mathbf{x})$, ..., $\mathbf{f}^n(\mathbf{x})$ multiple evaluation criteria functions that depict the strategy $\mathbf{x}$ into the vectors of target values $\mathbf{y}^1$, $\mathbf{y}^2$, ..., $\mathbf{y}^n$ of the target spaces of the players Y^1, Y^2, ..., Y^n. All players want to optimize the values of their multiple evaluation criteria functions. Number of criteria may be different for each player.

A dynamic model represents negotiation process, where individual time moments $t = 0, 1, 2, \ldots, T$ capture multi-round negotiation. The dynamic formulation of the problem captures the level of agreement or disagreement between the players. Reformulation of problems can be taken as searching for consensus through the exchange of information among players. At the time T the process is finalized by determining the trajectory to reach consensus. Dynamic negotiation process can be modeled as a progressive adjustment of the negotiating space until a one-element negotiating space is achieved.

A set of acceptable strategies is formulated for each player, where the strategies are acceptable with respect to specified aspiration levels. The aspiration levels $\mathbf{b}^i(t)$, $i = 1, 2, \ldots, n$, $t = 1, 2, \ldots, T$, correspond to opportunities for added values. At the start of the negotiations ($t = 0$) the set of acceptable strategies for player $i = 1, 2, \ldots, n$, has the form

$$X_i(0) = \{\mathbf{x}; \mathbf{x} \in X, \mathbf{f}^i(\mathbf{x}) \leq \mathbf{b}^i(0)\}, i = 1, 2, \ldots, n. \quad (9)$$

Then the negotiation space at the start of the negotiations ($t = 0$) is defined as an intersection of sets of the acceptable strategies of all players in negotiations

$$X_0(0) = \cap_{i=1}^{r} X_i(0) \quad (10)$$

Next negotiations take place over time periods t = 1, 2, ..., T. The negotiation process should be directed to a consensus strategy, to reach one-element negotiating space $X_0(t)$.

4.2. Concept of pressure

This concept of negotiation under pressure comes from the fact that each player decides under pressure subject to objective context with a set of internal and external pressures (Fiala 1999). A player is under pressure, if he wants to achieve a consensus in a time limit or in a situation where other players influence his behavior. The pressure affects decisions through a set of constraints that must be met. Thereafter, the pressure effects are shown in modifications of the set of constraints of the negotiation model. This will lead to a modification of the set of acceptable decisions for players and a modification of the negotiating space and may be directed to a consensus

Changes in aspiration levels for criteria functions due to the effects of pressures taking place at time periods t = 1, 2, ..., T, also modify the set of acceptable strategies

$$X_i(t) = \{\mathbf{x}; \mathbf{x} \in X, \mathbf{f}^i(\mathbf{x}) \leq \mathbf{b}^i(t)\}, i = 1, 2, \dots, n. \tag{11}$$

These changes are characterized by the following formula

$$\mathbf{b}^i(t) = \mathbf{b}^i(t-1) + \mathbf{p}^i(t). \tag{12}$$

Vector $\mathbf{p}^i(t)$ characterizes the adjustments of aspiration levels for the player i at time t *i*n comparison with aspiration levels at time t - 1. Vector $\mathbf{p}(t)$ describes the adjustments of all aspiration levels for all players at time t. So called trajectory of pressures is a continuous vector function $\mathbf{p}(t)$ defined on the interval [0, T] that is created by connection of vectors $\mathbf{p}(0)$, $\mathbf{p}(1)$, ..., $\mathbf{p}(T)$. The trajectory of pressures captures tactics of players in achieving the consensus, an equilibrium in supply chain.

5. Discussion and Conclusions

This paper proposes a general framework for equilibrium searching in supply chains. The problem-solving framework uses the network system with multiple units in and multiple evaluation criteria to structure the problem. Biform games are the basis of the process, combining cooperative and non-cooperative game instruments. The authors propose to classify biform games into sequential and simultaneous forms. The simultaneous form uses pressure negotiation concepts to achieve an equilibrium. The search for equilibrium is based on negotiating aspiration values of multiple evaluation criteria. The framework is flexible enough and allows an additional refinement of the supply chain equilibrium process, to extend the set of chain units by new and atypical units, to add additional evaluation criteria, and to include other solution concepts and approaches.

Standard multi-criteria decision techniques or state space searches using heuristic distance information from ideal criteria values can be used to search for criteria aspiration levels. The approach can also be enriched with multi-criteria De Novo optimization, where resource constraints in the chains are variable and the overall constraint is only a budget. New units (start-ups) can be included in the process of an equilibrium searching. The concept of co-opetition can bring new views into the analysis of supply chains, including adding new members such as competitors and complementors (competitors that create added value).

The model framework is open to complement other tools. Allocation mechanisms for the distribution of outputs can use other instruments, not only Shapley values, but also contracts (Fiala 2016a) and auctions (Fiala 2016b). Graph theory tools can be used to capture the complex structure of a modeled system with an environment in which units (nodes) formulate relations (edges) and flows to satisfy overall demand throughout the supply chain. The interconnection of these models and methods provides a suitable instrument for thorough supply chain analysis.

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