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A Comparison of Conditional and Unconditional VaR Models

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Abstract: This paper presents the empirical research on comparison of two different approaches for Value at Risk (VaR) measurement. The research objective is to compare the accuracy of out-of-sample VaR forecasts between conditional and unconditional models. We examine four unconditional models: Gaussian, alpha-stable, Normal Inverse Gaussian (NIG) and Generalized Pareto (GP) distributions and four conditional models: Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model with Gaussian and Student's t innovations, Exponentially Weighted Moving Average (EWMA) and conditional Extreme Value Theory (GARCH-EVT) approach. Calculations are performed on the basis of 5 world indices, 4 exchange rates and 4 commodity futures and the results are presented for left and right distribution tails. Backtesting methods indicate the GARCH-EVT as the model that outperforms all others.

Keywords: VaR; stable distribution; NIG; GPD; EVT; GARCH; EWMA; GARCH-EVT

JEL Classification: C22; G15; G17

1. Introduction

According to Basel Accords Value at Risk (VaR) plays a key role in calculation of regulatory capital for market risk. VaR is the maximum loss of a financial instrument or the entire portfolio X, that is not exceeded with a probability (confidence level) $1-\alpha$ in a given period of time. Formally, it is defined by the formula:

$$VaR_\alpha = -\sup\{q|P(X \le q) \le \alpha\}. \quad (1)$$

There are two methods which banks can use to measure VaR i.e. the standard approach (SA) and the internal models approach (IMA). The latter approach allows banks to use their own mathematical model to measure the market risk. The most popular and the simplest method is the variance-covariance method. Although, in practice, it is convenient to adopt the Gaussian paradigm, from a theoretical point of view, this is unacceptable. There are indisputable empirical properties of financial time series like leptokurtosis, the presence of fat tails of returns distribution, skewness and volatility clustering. Already in the sixties of the last century Benoit Mandelbrot (Mandelbrot 1963) rejected the normality of the distribution of returns and analyzed the alpha-stable distributions. Other distributions, such as generalized hyperbolic were the subject of other studies, including (Eberlein and Keller 1995; Barndorff-Nielsen 1997; Küchler et al. 1999). More sophisticated methods are derived from the Extreme Value Theory (EVT). EVT was applied for risk management in a number of publications e.g. (McNeil 1999; Embrechts et al. 1999; Gilli and Këllezi 2006).

In recent years, the conditional heteroskedasticity models, GARCH (Generalized Autoregressive Conditional Heteroskedasticity), introduced by Tim Bollerslev (Bollerslev 1986) have gained much popularity in risk management analysis. The conditional models can capture the dynamics and the most important properties of asset returns, e.g. volatility clustering and leptokurtosis. None of models, however, cannot predict exactly when the risk appears extreme and each has its strengths and weaknesses. Although the unconditional models use the strong assumption that returns are independent and equally distributed (i.i.d.), financial institutions often prefer unconditional risk forecast methods to avoid undesirable frequent changes in risk limits for traders and portfolio managers (Danielsson and de Vries 2000). Moreover, trading strategies, which are continuously updated, generate high transaction cost (Cotter 2007). On the other hand, the conditional GARCH methodology implies more volatile risk forecasts than the unconditional approach, which is desirable when short horizons of investment, like one day (Dowd 2005) or intraday (Danielsson and Payne 2000), are taken into account. A comprehensive review of Value at Risk methodologies present Abad et al. (2014).

Finally, it seems, that it is not possible to clearly identify the most appropriate model and following (Gilli and Këllezi 2006) the choice between conditional and unconditional model should depend ultimately on the period for the analysis and type of risk measure. In this paper, we examine several models of VaR measurement. The aim of this study is to compare the accuracy of VaR forecasts between conditional and unconditional models. Similar studies were already performed in i.a. (Kuester et al. 2006; Baran and Witzany 2011; Choi and Min 2011; Just 2014) but the set of considered models was different than in this paper. We take advantage of following unconditional distributions: Gaussian, alpha-stable, Normal Inverse Gaussian (NIG) and Generalized Pareto (GP) and conditional models: Exponentially Weighted Moving Average (EWMA), GARCH with Gaussian and Student's t innovations and conditional Extreme Value Theory (GARCH-EVT). Estimations are based on various markets including 5 stock indices, 4 exchange rates and 4 commodity futures from the period 2000 – June 2019. We do calculations of VaR for long and short investor position.

The remainder of this paper is organized as follows. Section 2 briefly summarizes different approaches to VaR measurement. Section 3 examines methods for testing the accuracy of VaR forecasts. In section 4 the data used in empirical study and the results of our research are described. Concluding remarks are provided in the final section.

2. Methodology

2.1. Unconditional Value at Risk

VaR for a long position is a minus quantile of the loss distribution:

$$VaR_{\alpha} = -F^{-1}(\alpha), \tag{2}$$

where F^{-1} is the inverse of cumulative distribution function of returns, F. For short position it is a $1-\alpha$ quantile of the distribution:

$$VaR_{1-\alpha} = F^{-1}(1-\alpha). \tag{3}$$

We briefly characterized the distributions which are used in our studies.

Gaussian (normal) distribution is characterized by only two parameters, mean, $\mu \in \mathbb{R}$ and standard deviation, $\sigma > 0$. The probability density function is of the form:

$$f_{NORM}(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}. \tag{4}$$

A lot of well-known desirable properties of the normal distribution make it the most useful distribution in finance. However, its tails are too thin to precisely measure a quantile of empirical distribution of returns. The tails decay very quickly (faster than exponentially), but it is still in use in practice to measure VaR. In this study it is used rather as a benchmark only.

Alpha-stable distributions are a broad family of probability distributions which can capture skewness and heavy tails. Alpha-stable distributions contain the Cauchy, Gaussian and Lévy distributions. The class was introduced by Paul Lévy in 1924 (Lévy 1924). It was the first alternative to Gaussian distribution in finance (Mandelbrot 1963), and now is widespread in risk management, forecasting and econometrical analysis (Bradley and Taqqu 2003; Kabasinskas et al. 2009; Nolan 2009; Rachev et al. 2009). In the general case, the analytic form of distribution function does not exist, therefore it is defined by characteristic function:

$$\mathbb{E}[exp\,(itX)] = \begin{cases} exp\left(-c^{\alpha}|t|^{\alpha}\left(1 - i\beta sign(t) tg\left(\frac{\pi\alpha}{2}\right)\right) + i\tau t\right), & \alpha \neq 1, \\ exp\left(-c|t|\left(1 + i\beta\frac{2}{\pi} sign(t) log\,(|t|)\right) + i\tau t\right), & \alpha = 1. \end{cases} \quad (5)$$

The stable distribution is defined by four parameters. The most important parameter $\alpha \in (0,2]$ is called stability index, parameter $\beta \in [-1,1]$ is a measure of skewness, $\tau \in \mathbb{R}$ is a position and $c > 0$ is a scale parameter. Parameter α is responsible for the thickness of the tail. The lower its value is, the thicker the tail is. For $\alpha \in (0,2)$ the second moment of the distribution does not exist, but for $\alpha = 2$ it is obtained the Gaussian distribution as a limit case. Alpha-stable family of distributions is closed under linear transformations, which means that the linear combination of random variables with alpha-stable distribution with the same index of stability α, also has alpha stable distribution with that index.

*Normal Inverse Gaussian (*NIG*) distribution* is a member of the broader class of distributions called generalized hyperbolic (GH) introduced in (Barndorff-Nielsen 1977). This distribution is widely applied in financial economics for modeling unconditional and conditional return distribution (Haas and Pigorsch 2009). Log density of NIG is concave in an interval around zero and convex in the tails. It is a typical property of financial returns which exhibits tail behavior that is heavier than log-linear (Barndorff-Nielsen 1997). The density of the NIG distribution is given by

$$f_{NIG}(x) = \frac{\alpha\delta K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}} exp\left(\delta\sqrt{\alpha^2-\beta^2} + \beta(x-\mu)\right), \quad (6)$$

where: α and β are shape parameters fulfilling condition $0 < |\beta| \leq \alpha$, and $\mu \in \mathbb{R}$, $\delta > 0$ are respectively position and scale parameters. K_1 is modified Bessel function of the third kind with index equal to one.

Generalized Pareto (GP) *distribution* is one of two key distributions of Extreme Value Theory. The role of GP distribution in EVT is as a natural model for the excess distribution over high threshold. This is called Peaks over Threshold (POT) approach and it is based on Pickands-Balkema-de Haan Theorem (Balkema and de Haan 1974). For high threshold u, the conditional distribution function:

$$F_u(x) = P(X - u \leq x | X > u) = \frac{F(x+u) - F(u)}{1 - F(u)}, \quad (7)$$

converges to a generalized Pareto distribution. GP distribution is as follows

$$G_{\xi,\beta}(x) = \begin{cases} 1 - \left(1 + \xi\frac{x}{\beta}\right)^{-\frac{1}{\xi}}, & \xi \neq 0, \\ 1 - exp\left(-\frac{x}{\beta}\right), & \xi = 0, \end{cases} \quad (8)$$

where: $\beta > 0$, $x \geq 0$ for $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ for $\xi < 0$. The shape parameter, ξ divides distributions into three classes. Heavy tail distributions (e.g. alpha-stable, Student's *t*) have $\xi > 0$ (Fréchet domain of attraction). Thin tail distributions (e.g. Gaussian, log-normal) have $\xi = 0$ (Gumbel domain of attraction). Distributions with finite right endpoint have $\xi < 0$ (Weibull domain of attraction). The unconditional cumulative distribution function of returns one obtain rearranging (7)–(8):

$$F(x) = \left(1 - F(u)\right)G_{\xi,\beta}(x)(x - u) + F(u), \quad x > u. \quad (9)$$

Replacing $F(u)$ by $\hat{F}(u) = 1 - N_u/n$, where N_u is a number of exceedances over threshold u and n is number of returns we obtain:

$$\hat{F}(u) = 1 - \frac{N_u}{n}\left(1 + \xi\frac{(x-u)}{\beta}\right)^{-\frac{1}{\xi}}. \quad (10)$$

VaR for short position (right tail) is easy obtainable by inverting the above equation:

$$VaR_{1-\alpha} = u + \frac{\beta}{\xi}\left(\left(\frac{n}{N_u}\alpha\right)^{-\xi} - 1\right). \quad (11)$$

2.2. Conditional Value at Risk

We assume following process of returns:

$$r_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim i.i.d.(0,1), \tag{12}$$

where σ_t is conditional volatility and the innovations ε_t have distribution F.
Conditional VaR computed for long position is as follows:

$$VaR_\alpha = -\sigma_t(1)F^{-1}(\alpha), \tag{13}$$

where F^{-1} is the inverse of cumulative distribution function F and $\sigma_t(1)$ is one step ahead forecast of conditional standard deviation. For short position it is equal to:

$$VaR_{1-\alpha} = \sigma_t(1)F^{-1}(1-\alpha). \tag{14}$$

We briefly characterized three such models which will be used in our study.

GARCH model. There is a broad family of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models but the most popular is GARCH(1,1) (Bollerslev 1986):

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{15}$$

where: $\omega, \alpha, \beta > 0, \alpha + \beta < 1$.
The parameters α and β represent the adjustments to past market shocks and volatility respectively.

Exponentially Weighted Moving Average (EWMA) volatility model takes into account the property that the influence of any observation in financial time-series declines over time at the stable rate $\lambda > 0$. The model was adopted in 1994 by U.S. investment bank JP Morgan in RiskMetrics methodology. The variability in this model are determined by formula:

$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2, \tag{16}$$

where $0 < \lambda < 1$.
Parameter λ is not estimated, but it is taken at the level of 0.94 for the daily data, and at the level of 0.97 for monthly data. It determines the ease use of this method in practice. In fact EWMA model belongs to the family of GARCH models called IGARCH (1,1).

Conditional EVT (GARCH-EVT) model is a concept of McNeil and Frey (2000) to VaR modeling by extending the EVT framework to dependent time-series. In this model, we fit the GP distribution parameters to standardized residuals e_t of GARCH model and then calculate VaR for short position as follows:

$$VaR_{1-\alpha} = \sigma_t(1) VaR_{1-\alpha}(e_t), \tag{17}$$

where: $\sigma_t(1)$ is one step ahead forecast of conditional standard deviation in GARCH model and $VaR_{1-\alpha}(e_t)$ is calculated from (11) but for the standardized residuals e_t of GARCH model.

3. Backtesting Methods

Backtesting procedure performs the comparison of Value at Risk estimations to actual losses of the considered assets. The accuracy of the model is assessed here on the basis of the number of returns exceeding VaR. Calculating the VaR at a tolerance level α, it is required, that the percentage of the VaR exceeded by empirical returns to the all ones in the sample would equal to α. If exceedances' percentage is higher than assumed, this model underestimates the risk, otherwise the VaR model is too conservative, and the actual risk is lower than the model shows. The most used backtesting tool is the Kupiec's test (Kupiec 1995) known as the proportion of failures test. This test verifies if the actual number of VaR exceedances is equal to α. The test statistic is defined as follows:

$$LR\_UC = 2\left(log\left(\left(\frac{T_1}{T_0+T_1}\right)^{T_1}\left(1-\frac{T_1}{T_0+T_1}\right)^{T_0}\right) - log(\alpha^{T_1}(1-\alpha)^{T_0})\right), \tag{18}$$

where: T_1– the number of VaR exceedances, T_0– the number of unexceeded VaR. Under the true null hypothesis $LR\_UC \sim \chi^2(1)$.

Christoffersen's test (Christoffersen 1998) is more sophisticated statistical test and aside from the number of exceptions additionally checks independence of VaR exceedances. More precisely it verifies if the current exception is independent on the exception appearance on the previous day. The test statistic is defined as follows:

$$LR\_CC = 2log\left(\left(\frac{T_{01}}{T_{01}+T_{00}}\right)^{T_{01}}\left(1-\frac{T_{01}}{T_{01}+T_{00}}\right)^{T_{00}}\left(\frac{T_{11}}{T_{10}+T_{11}}\right)^{T_{11}}\left(1-\frac{T_{11}}{T_{10}+T_{11}}\right)^{T_{10}}\right)+ \\ -2log(\alpha^{T_{01}+T_{11}}(1-\alpha)^{T_{00}+T_{10}}), \quad (19)$$

where: T_{ij} – the number of days when exception j occurred assuming that exception i occurred on the previous day (1 if violation occurs, 0 if no violation occurs). Under the true null hypothesis $LR\_CC \sim \chi^2(2)$.

The Christoffersen and Pelletier's test analyses if the number of days between exceedances is independent over time (Christoffersen and Pelletier 2004). Under the null hypothesis the duration of time between VaR violations should have no memory and mean duration of $1/\alpha$. The test is based on the Weibull distribution, which is the memory free distribution. Here the Weibull distribution with parameter $b = 1$ is used. The distribution is of the form:

$$f(d) = a^b b d^{b-1} exp\{-(ad)^b\}, \quad (20)$$

where d is the number of days between two violations of VaR. Under the null hypothesis of independence the likelihood is as follows:

$$L(\alpha) = \prod_{t=1}^{T_1-1}\left(\alpha\, exp(-\alpha\, d_t)\right), \quad (21)$$

where T_1 is the number of days in which a violation is occurred. The likelihood ratio test statistic is thus:

$$LR\_UD = 2\left(logL(\hat{a}) - logL(\alpha)\right). \quad (22)$$

Under the true null hypothesis $LR\_UD \sim \chi^2(1)$. We refer to (Christoffersen and Pelletier 2004) for details of the test.

The loss function is a goodness-of-fit measure for VaR calculation. The loss function for a given α is defined as follows (Gonzales-Rivera et al. 2004):

$$Q = P^{-1}\sum_{t=R}^{T}(\alpha - I_{t+1}(\alpha))(r_{t+1} + VaR_{\alpha}(r_t)), \quad (23)$$

where: $I_{t+1}(\alpha) = 1$ for $r_{t+1} < -VaR_{\alpha}(r_t)$ and $I_{t+1}(\alpha) = 0$ otherwise, P – the prediction period, R – the estimation period. A lower Q value means a better goodness of fit.

4. Results and Discussion

To test the forecasting performance of examined VaR models we selected 4 currencies, 5 stock indices and 4 commodities: USD/EUR, USD/GBP, USD/JPY, USD/PLN, S&P500 (SPX), FTSE100 (UKX), NIKKEI225 (NKX), ATHEX COMP (ATX), WIG20, GOLD (GC.F), SILVER (SI.F), CRUDE OIL (CL.F), NATURAL GAS (NG.F). The data comprises of daily price levels of the chosen assets from the beginning of 2000 up to 30th June, 2019. The data set was obtained from the financial stock news website (stooq.pl). We use log-returns (as a percentage) in our calculations. We examine the VaR forecasts at two significance levels, i.e. 1% and 5% for both the long and the short investor position. For the sake of brevity we present the results only for 5% VaR in the tables 1–8. Results for 1% VaR are available from the authors on the request. For all considered models, we allow the model parameters to change over time. Using rolling windows of size 500 we daily update the model parameters estimates and calculate VaR forecasts for the next trading day. We calculate VaRs using the following unconditional models: Gaussian distribution (NORM), stable distribution (STAB), Normal Inverse Gaussian distribution (NIG) and Generalized Pareto distribution (GP) assuming arbitrary threshold level of 90% (i.e. the largest 10% of positive and negative returns are considered as the extreme observations). We also use conditional models like: EWMA, GARCH with Gaussian (GARCH-NORM) and Student's *t* (GARCH-*t*) innovations and McNail and Frey GARCH-EVT model with Gaussian innovations – assuming the threshold in the same way as in the unconditional model.

Echaust (2018) and Echaust and Just (2020) considered GARCH-EVT model with optimal tail selection and updated the optimal tail fraction for each moving window of observations. They did not find the improvement of VaR forecasts accuracy with reference to a constant threshold approach. In order to verify the effectiveness of examined models, the expected (ET) and the actual (T_1) number of VaR exceedances are determined and Kupiec's, Christoffersen's and Christoffersen and Pelletier's tests are applied to verify a correctness of models. Additionally, the tests are supplemented by the loss function.

Assessing the quality of the estimated VaRs for the analyzed assets, based on the Kupiec's test it can be concluded that the worst results are obtained for the unconditional model with a normal distribution. Especially for 1% VaR the Gaussian distribution has too thin tails and too many exceedances of VaR appear. Such a situation takes place in 11 out of 13 analyzed assets for both left and right tails (significance level of 5%). It is possible to get an improvement of the results using NIG or GP distributions, which allow to capture the fat tails property of the empirical distribution. They measure the number of exceedances very accurately for 5% VaR but for 1% VaR the models fail in 4–5 out of 13 cases. In these cases, the number of VaR's exceedances, estimated by using unconditional models, exceeds the acceptable level. It means, that VaRs determined by using these methods are underestimated. Since it is not possible to estimate the stable distribution parameters for each window of observation, therefore these results are placed in the table 8 only for two assets i.e. SPX and SI.F. As is typical for this distribution, unlike the other unconditional models, this model overestimates the high quantiles (1% and 99% here). The accuracy of conditional models vary depending on the type of model which is used to measure the VaR. EWMA model produces VaR forecasts seriously inaccurate and the number of exceedances is much higher than the expected level. For the tolerance level of 1% the model generates the worst VaR forecasts between all considered models (except the unconditional model with a normal distribution). The GARCH models perform much better, but surprisingly they both are outperformed by unconditional NIG and GP distributions for 5% VaR. The improvement of the quality of VaR estimations is achieved for the GARCH-EVT model. This is only one model that produces accurate VaR forecasts for both significance levels and for all considered assets.

The Christoffersen's and Christoffersen-Pelletier's tests focus on independence of VaR exceedances instead of their number only. Since unconditional models do not account for volatility clustering none of them is able to produce i.i.d. VaR violations. In almost all considered cases, we reject the null hypothesis, which states that the VaR exceedances are independent over time. The exception is the Christoffersen's test which fails to reject independence in some cases for each distribution. The NIG and GP distributions perform the best between unconditional models and they seem to be good models in 4 out of 13 for left tail and in 6 out of 13 cases for 5% VaR, and in half of cases for 1% VaR. Conditional models perform significantly better. The volatility models, GARCH and GARCH-EVT, in most of the analyzed cases, can capture stylized facts about financial time series like volatility clustering and leptokurtosis, as well as the skewness of the distribution (GARCH-EVT). The worst model is EWMA which fails in almost half of the cases. The GARCH-EVT model occurs to be the most preferable for both considered tolerance levels and for both the right and the left tails. For this model the Christoffersen's test rejects the null hypothesis only once for the left and the right tail in the 5% VaR case and twice for the left tail in the 1% VaR case. The Christoffersen-Pelletier's test indicates the dependence of the number of days between following exceedances two times for the left tail and five times for the right one in the 5% VaR case, and only once for the left tail in the 1% VaR case.

Loss function achieves approximately the same values for all conditional models and the same for all unconditional models. However, the values obtained from conditional methods are lower than those from unconditional models, implying that they offer higher accuracy than unconditional models.

Table 1. Backtesting VaR estimation under normal distribution.

NORM		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
USD/EUR	227	207	2.07	9.03*	16.25**	0.066	219	0.37	6.58*	36.08**	0.068
USD/GBP	227	214	0.87	4.20	17.83**	0.061	232	0.09	12.82**	25.88**	0.068
USD/JPY	227	209	1.69	8.28*	32.65**	0.073	199	4.01*	7.05*	11.65**	0.068
USD/PLN	225	190	6.15*	16.46**	27.02**	0.09	240	0.98	22.56**	42.12**	0.098
SPX	220	248	3.58	29.81**	93.12**	0.143	192	3.94*	10.06**	70.86**	0.128
UKX	221	219	0.02	26.50**	81.23**	0.137	196	3.14	10.11**	58.02**	0.129
NKX	214	206	0.32	10.98**	61.20**	0.18	165	12.80**	15.85**	39.77**	0.156
ATH	216	197	1.88	23.94**	59.30**	0.21	166	13.36**	19.22**	31.39**	0.197
WIG20	218	211	0.31	35.06**	53.08**	0.162	205	0.96	2.20	15.03**	0.151
GC.F	222	233	0.56	7.76*	8.84**	0.134	190	5.12*	5.79	7.17**	0.117
SI.F	222	221	0.01	11.38**	6.41*	0.241	188	5.80*	5.80	18.97**	0.198
CL.F	222	240	1.48	11.57**	44.42**	0.261	213	0.40	7.20*	39.95**	0.241
NG.F	222	159	20.83**	23.73**	33.81**	0.34	191	4.79*	7.41*	43.65**	0.381

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

Table 2. Backtesting VaR estimation under NIG distribution.

NIG		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
USD/EUR	227	218	0.45	5.53	13.02**	0.067	224	0.07	5.44	34.05**	0.067
USD/GBP	227	245	1.37	3.92	14.47**	0.061	237	0.40	11.87**	20.97**	0.068
USD/JPY	227	233	0.12	6.53*	27.10**	0.073	223	0.11	5.64	19.76**	0.068
USD/PLN	225	224	0.01	14.61**	23.85**	0.091	237	0.62	19.09**	41.92**	0.098
SPX	220	246	3.10	32.42**	96.40**	0.142	237	1.33	6.41*	86.95**	0.129
UKX	221	225	0.07	28.96**	74.77**	0.136	224	0.04	6.14*	54.62**	0.129
NKX	214	208	0.18	10.36**	59.57**	0.18	197	1.47	7.67*	35.14**	0.155
ATH	216	204	0.76	30.09**	55.77**	0.208	200	1.33	15.75**	24.50**	0.196
WIG20	218	212	0.24	34.51**	53.67**	0.162	234	1.07	1.62	11.13**	0.152
GC.F	222	230	0.29	3.33	7.10**	0.133	226	0.07	1.38	6.02*	0.117
SI.F	222	222	0.00	14.73**	5.69*	0.241	243	2.01	2.25	12.23**	0.196
CL.F	222	238	1.17	11.72**	47.63**	0.26	235	0.78	12.05**	38.56**	0.242
NG.F	222	205	1.41	3.52	24.90**	0.335	203	1.77	5.08	37.88**	0.381

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

Table 3. Backtesting VaR estimation under GP distribution.

GP		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
USD/EUR	227	210	1.51	7.92*	14.53**	0.067	215	0.78	5.09	36.27**	0.068
USD/GBP	227	236	0.32	0.97	10.88**	0.061	230	0.03	15.12**	26.17**	0.068
USD/JPY	227	230	0.02	6.97*	19.97**	0.072	232	0.08	3.23	15.29**	0.068
USD/PLN	225	220	0.14	15.90**	29.76**	0.091	233	0.27	12.46**	38.01**	0.097
SPX	220	225	0.11	33.86**	102.02**	0.142	228	0.30	6.90*	82.63**	0.129
UKX	221	223	0.02	27.29**	80.46**	0.137	220	0.01	8.28*	55.97**	0.130
NKX	214	218	0.08	5.32	42.84**	0.179	202	0.73	7.45*	37.97**	0.155
ATH	216	205	0.64	26.97**	61.09**	0.208	209	0.27	18.14**	30.61**	0.197
WIG20	218	219	0.00	36.36**	51.94**	0.163	211	0.31	1.14	15.12**	0.153
GC.F	222	221	0.01	4.22	7.78**	0.133	225	0.04	0.24	7.67**	0.116
SI.F	222	226	0.07	17.51**	7.02**	0.241	235	0.78	0.99	12.86**	0.197
CL.F	222	237	1.03	10.28**	41.57**	0.259	232	0.46	7.86*	32.11**	0.241
NG.F	222	221	0.01	0.85	22.49**	0.336	215	0.24	3.13	43.10**	0.380

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

Table 4. Backtesting VaR estimation under EWMA model.

EWMA		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
USD/EUR	227	262	5.15*	5.21	8.13**	0.062	250	2.20	2.34	1.67	0.062
USD/GBP	227	239	0.59	1.21	5.42*	0.058	263	5.53*	6.32*	3.59	0.062
USD/JPY	227	234	0.17	0.26	0.12	0.068	218	0.45	1.13	3.61	0.065
USD/PLN	225	224	0.01	0.77	4.57*	0.082	264	6.63*	12.34**	0.02	0.089
SPX	220	246	3.10	3.22	0.99	0.12	226	0.17	5.37	13.38**	0.103
UKX	221	277	13.77**	26.89**	0.35	0.118	219	0.02	8.60*	16.84**	0.104
NKX	214	248	5.41*	5.61	1.24	0.161	221	0.24	2.92	2.74	0.136
ATH	216	248	4.66*	21.97**	9.71**	0.192	217	0.00	2.86	1.10	0.182
WIG20	218	233	0.93	5.40	4.93*	0.149	242	2.47	2.48	16.20**	0.142
GC.F	222	248	3.07	3.36	4.12*	0.126	219	0.05	6.15*	13.73**	0.112
SI.F	222	258	5.82*	6.11*	6.60*	0.226	205	1.42	4.17	0.49	0.186
CL.F	222	258	5.82*	8.27*	0.01	0.234	216	0.18	5.88	16.12**	0.21
NG.F	222	218	0.08	0.90	2.49	0.312	268	9.42**	10.17**	5.05*	0.364

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

Table 5. Backtesting VaR estimation under GARCH model with Gaussian innovations.

GARCH -NORM		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
USD/EUR	227	216	0.66	0.72	2.38	0.062	208	1.87	2.12	0.57	0.063
USD/GBP	227	215	0.75	1.29	2.11	0.058	243	1.07	1.07	0.49	0.062
USD/JPY	227	211	1.34	1.72	0.06	0.069	196	4.91*	6.68*	5.46*	0.065
USD/PLN	225	189	6.52*	6.52*	1.48	0.082	237	0.62	2.29	0.03	0.089
SPX	220	239	1.66	1.66	0.01	0.119	205	1.12	3.96	14.09**	0.103
UKX	221	252	4.33*	5.30	0.83	0.118	194	3.67	11.36**	13.56**	0.103
NKX	214	229	1.08	1.22	0.52	0.16	184	4.65*	4.65	2.55	0.134
ATH	216	203	0.89	7.62*	4.73*	0.19	191	3.25	4.10	1.31	0.182
WIG20	218	203	1.25	6.79*	6.09*	0.149	226	0.24	1.63	10.00**	0.142
GC.F	222	219	0.05	1.72	3.31	0.125	199	2.62	6.30*	1.63	0.112
SI.F	222	237	1.03	3.30	1.62	0.229	181	8.52**	12.10**	0.01	0.187
CL.F	222	231	0.37	0.37	0.29	0.234	185	6.90**	7.34*	6.21*	0.211
NG.F	222	187	6.14*	6.67*	2.03	0.313	235	0.78	1.35	2.31	0.363

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

Table 6. Backtesting VaR estimation under GARCH model with Student's *t* innovations.

GARCH-*t*		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
USD/EUR	227	225	0.04	0.38	1.90	0.062	205	2.49	2.50	0.11	0.063
USD/GBP	227	219	0.35	0.61	1.18	0.058	238	0.49	0.70	0.05	0.062
USD/JPY	227	224	0.07	0.18	0.20	0.068	208	1.87	3.48	2.85	0.065
USD/PLN	225	199	3.37	3.55	0.92	0.082	244	1.58	5.01	0.10	0.089
SPX	220	247	3.34	3.40	0.29	0.119	212	0.32	3.87	17.65**	0.103
UKX	221	254	4.90*	5.74	1.16	0.118	202	1.81	10.69**	14.12**	0.103
NKX	214	232	1.54	1.77	1.31	0.161	188	3.47	3.85	2.23	0.134
ATH	216	219	0.03	6.60*	3.21	0.19	196	2.08	7.70*	1.97	0.181
WIG20	218	210	0.39	10.64**	7.11**	0.15	238	1.70	3.15	13.80**	0.141
GC.F	222	239	1.32	2.09	3.01	0.125	220	0.02	6.26*	4.30*	0.111
SI.F	222	259	6.14*	9.30**	2.90	0.228	209	0.83	2.72	0.43	0.185
CL.F	222	235	0.78	0.99	0.18	0.234	187	6.16*	6.68*	4.97*	0.211
NG.F	222	203	1.77	3.21	0.19	0.313	251	3.82	4.22	2.05	0.362

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

Table 7. Backtesting VaR estimation under GARCH-EVT model.

GARCH-EVT		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
USD/EUR	227	209	1.69	1.91	0.88	0.062	218	0.45	0.68	1.12	0.063
USD/GBP	227	228	0.00	1.26	4.71*	0.058	221	0.20	0.21	0.30	0.062
USD/JPY	227	228	0.00	0.60	0.03	0.069	220	0.29	2.89	3.75	0.065
USD/PLN	225	210	1.13	2.19	0.23	0.082	226	0.00	0.65	0.04	0.089
SPX	220	223	0.04	0.71	0.21	0.119	217	0.05	4.16	12.44**	0.104
UKX	221	226	0.11	1.83	0.34	0.119	234	0.77	11.83**	17.08**	0.104
NKX	214	220	0.17	0.73	1.19	0.16	205	0.41	0.49	4.13*	0.134
ATH	216	210	0.20	8.55*	2.28	0.19	222	0.15	0.75	3.11	0.182
WIG20	218	224	0.12	4.81	4.13*	0.15	220	0.01	1.84	20.65**	0.142
GC.F	222	222	0.00	3.07	3.72	0.126	217	0.12	4.12	3.07	0.111
SI.F	222	224	0.02	1.96	1.64	0.229	214	0.32	5.76	0.95	0.188
CL.F	222	227	0.11	0.29	0.19	0.234	210	0.71	1.15	5.43*	0.211
NG.F	222	220	0.02	0.41	1.04	0.312	229	0.23	1.03	2.62	0.365

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

Table 8. Backtesting VaR estimation under stable distribution.

STAB		Lower tail, VaR 0.05					Upper tail, VaR 0.95				
Asset	**ET**	**T1**	**UC**	**CC**	**UD**	**Loss**	**T1**	**UC**	**CC**	**UD**	**Loss**
SPX	220	229	0.37	29.78**	104.63**	0.142	237	1.33	8.93**	81.71**	0.130
SI.F	222	227	0.11	19.28**	8.56**	0.242	238	1.17	1.18	11.13**	0.197

Note: ET (T1) – expected (actual) number of VaR violations, UC – Kupiec's test statistic, CC – Christoffersen's test statistic, UD – Christoffersen and Pelletier's test statistic, Loss – loss function Q described as in Gonzalez-Rivera et al. (2004), p-value<0.01 (**), 0.01<p-value<0.05 (*).

5. Conclusions

The aim of this paper has been to evaluate how well unconditional and conditional models perform in estimating and forecasting a VaR measure. We employ four unconditional models i.e. Gaussian, NIG, GP, and stable distributions and four conditional models i.e. EWMA, GARCH with Gaussian and Student's *t* innovations and GARCH-EVT models. Definitely worse VaR estimations are obtained for unconditional models, and especially poor for the Gaussian distribution. An improvement of VaR accuracy is obtained for VaR calculated from NIG and GP distributions, which can better model extreme returns. However, these estimations are still not good. We have shown that unconditional models usually underestimate the VaR for a small tolerance VaR level (1%). Even if they provide the VaR estimates, for which the number of their exceedances by the empirical returns is in line with the assumed level, the exceedances are not independent over time. They provide the stable estimates of model parameters and do not update quickly when the volatility changes. The majority of VaR exceedances occurred during periods of high volatility, when VaR values were estimated based on periods of low volatility. However, in periods of low volatility, they occurred after periods of high volatility and the VaRs have not been exceeded. Conditional models are deprived of this defect. The clustering of returns volatility is well captured by conditional models like GARCH-*t* and GARCH-EVT. Especially the latter model should be distinguished because of good VaR estimations regardless of considered assets, the level of tolerance and investor position (long and short).

The presented results concern the verification of VaR models in a very short, one-day time horizon. When we analyze the accuracy of VaR forecasts in a longer period of time e.g. 10 days, these results may differ significantly from those ones presented in this work. This problem will be considered in the authors' future work.

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