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Dynamic Containers Loading Problem

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Abstract. This paper deals with the container loading problem where parcels need to be loaded in the certain time interval. The aim is to load all parcels while minimizing the number of containers. The capacity of the container can´t be exceeded. Every parcel has its given volume. At the same time, this paper also deals with container´s time schedule, therefore with planning the time when the container will be used for parcel transport. Then the mathematical model is proposed and tested. In practice, container loading goes continuously, therefore heuristic solution method on an unlimited time interval is proposed for this problem. Information about parcels is available gradually. The method is illustrated with a numerical example.

Keywords: Container Loading Problem, Heuristics, Integer Programming.

1 Introduction

Freight transport represents a continuous process of shifting goods from a manufacturer to customers through chain´s warehouses. Various means of transport and containers are used. The aim is to minimize transport costs, which is related to optimizing the type of mean of transport, route of transport and also with optimizing vehicle´s or container´s capacity.

This paper is focused on a certain part of optimizing the transport of goods, namely on the container loading optimization, where the requirement for transporting goods is defined and the time when the transport of the goods must be realized is within a certain time interval. Demand of loading in the time i will be denoted as parcel i. The volume (weight, volume, load area…) and time interval, during which the shipment should be loaded, is known for each parcel. This information is available for a certain period of time. The aim is to minimize the number of used containers for this period. Every parcel must be loaded, and the capacity of containers is limited. When solving this task for a finite period of time, ineffective container loading occurs at the end of this period. Therefore, it is necessary to solve this task within an infinite time horizon, gradually gaining information about future parcels.

In literature, this task is solved as static, but the problem of locating parcels inside the container is solved as well. Bortfeldt and Waschermake review of constraints that may be respected in container loading [1]. In literature, this problem is called truck loading problem. In the scientific literature, there are several papers about the truck loading problem. Their contents are different from the problems solved in this article. Many of these sources solve the problem of cyclic drives of the trucks loaded with different types of products, e. g. the oil products, so that the time interval between two deliveries is maximal [3]. Another author [2] proposed a way of periodical loading of carriers with more compartments on the load area where the compartments have the various capacity and different types of goods are considered. The three-dimensional packing problem in which rectangular boxes must be effectively placed into containers is presented in [3]. In this paper we review the research focusing on the mathematical models providing the exact solution and heuristic algorithms giving approximate solution.

The problem, that is the subject of this paper, concerns time period <1,*T*>, during which the parcels should be loaded into containers with the same capacity *W*. We assume that container can be loaded in discrete time 1,2…,*T*. No more than one container can be loaded at every time. It is necessary to load the parcels into the container during a given time interval. The size of parcel i.e. occupied part of vehicle´s capacity is also known. The aim is to load all parcels when using a minimum number of containers.

2 Mathematical Model

Container loading problem concerns time period <1,*T*>. Parcel available in time $i \in I=\{1,2,\ldots,T\}$ should be loaded in time $i, i+1, \ldots, i+d_i$ into the container with capacity W. Let´s denote the size of this parcel w_i and assume, that $w_i \leq W$.

Parameters of the model are:

w_i size of the parcel, which is available for loading in the time i and should be loaded in time $<i, i+d_i>$, where $i+d_i \leq T$,

W capacity of containers.

Variables of the model are:

x_{ij} binary, equals 1 if the parcel i is loaded in the time j into the container,

y_j binary, equals 1 if the container is loaded in the time j.

Mathematical model of loading problem (basic model BM)

$$f(y) = \sum_{j=1}^{T} y_j \rightarrow min \quad (1)$$

$$\sum_{i=1}^{T} w_i x_{ij} \leq W y_j \text{ , } j = 1,2,\ldots,T \quad (2)$$

$$x_{ij} = 0, \ i = 1,2,\ldots,T, \ j < i, \quad (3)$$

$$x_{ij} = 0, \ i = 1,2,\ldots,T, \ j > min\{i + d_i, T\}, \ i = 1,2,\ldots,T \quad (4)$$

$$\sum_{j=1}^{T} x_{ij} = 1 \text{ , } i = 1,2,\ldots,T \quad (5)$$

$$x_{ij}, \ y_j \ binary, i,j = 1,2,\ldots,T. \quad (6)$$

The objective function (1) minimizes the number of used containers. Constraint (2) states that the container can´t be loaded more than its given capacity. If there is no available container in the time j, it is not possible to load parcel in this time. Constraints (3) and (4) determine that parcel can´t be loaded outside the time interval $<i,\ i+d_i>$. Equation (5) assures that every parcel must be loaded.

Remark. The solution of the mathematical model concerns the finite time interval. In practice, container loading problem is continuous, thus it goes on to infinity. If we use the solution from the mathematical model, then the parcels available at the end of the interval $<1,T>$ will require container utilization although its capacity won´t be used effectively. It would be appropriate to load these parcels along with parcels that will be available in terms T+1 T+2,…, thereby better utilization of container´s capacity will be achieved.

3 Infinite Time Interval Problem

Now we will be solving container loading problem in an infinite time interval, thus from 1 to infinity. Parcels which are available in the interval $<T+1,2T>$ are known at the time T, parcels which have to be loaded in $<2T+1,3T>$ are known at $2T$, etc. This continual container loading problem will be solved with a series of models. Let´s denote the first of them as first model FM. It is a modified model (1)-(6), which solves the parcels from the interval $<1,T>$. Parcels from the interval $<T+1,2T>$ will be solved by subsequent model SM. We can continue this way forward gradually solving SM models for subsequent intervals.

For simplicity let´s assume that $d_i = 2$ for every i, thus the interval length within it is possible to load the parcel is 3 time periods, therefore the parcel i has to be loaded in the time i, i+1 and i+2.

It is not necessary to load the parcels w_{T-1} and w_T in time T-1 or in time T, it can be loaded later in time T+1 or T+2 respectively. Because of that, we will move the problem of its loading them to the subsequent model SM. Equation (5) won´t be valid for these parcels.

Mathematical model of the loading problem in the initial time interval (first model FM)

$$f(y) = \sum_{j=1}^{T} y_j \rightarrow min \tag{1a}$$

$$\sum_{i=1}^{T} w_i x_{ij} \leq W y_j\ ,\ j = 1,2,\dots,T \tag{2a}$$

$$x_{ij} = 0,\ \ i = 1,2,\dots,T,\ \ j < i, \tag{3a}$$

$$x_{ij} = 0,\ \ i = 1,2,\dots,T,\ \ j > min\{i+2,T\},\ \ i = 1,2,\dots,T \tag{4a}$$

$$\sum_{j=1}^{T} x_{ij} = 1\ ,\ i = 1,2,\dots,T-2 \tag{5a}$$

$$x_{ij,} \;\; y_j \;\; binary, i, j = 1,2, \dots, T. \tag{6a}$$

We obtain optimal solution y^* and x^* by solving model FM. Subsequently, we will solve the SM model using available information about parcels from the interval <T+1,2T> and results from first model FM.

Mathematical model of the loading problem in the subsequent time interval (subsequent model SM)

$$f(y) = \sum_{j=T+1}^{2T} y_j + (1 - y_{T-1}^*) y_{T-1} + (1 - y_T^*) y_T \rightarrow min \tag{1b}$$

$$\sum_{i=T-1}^{2T} w_i x_{ij} \leq W y_j \;, \;\; j = T-1, T, \dots, 2T-2 \tag{2b}$$

$$x_{ij} = 0, \;\; i = T-1, T, \dots, 2T, j < i, \tag{3b}$$

$$x_{ij} = 0, \;\; i = 1,2, \dots, T, \; j > min\{i+2, 2T\}, \; i = T-1, T, \dots, 2T \tag{4b}$$

$$\sum_{j=T-1}^{2T} x_{ij} = 1 \;, \;\; i = T-1T, \dots, 2T-2 \tag{5b}$$

$$x_{ij,} \;\; y_j \;\; binary, i, j = T-1, T, \dots, 2T. \tag{6b}$$

The subsequent model SM can be applied not only for the time interval <T+1,2T>, but for all subsequent intervals: <2T+1,3T>, <3T+1,4T>,…

4 Numerical Experiments

In the following example, we will illustrate the results from the problem described above. In Table 1 and Table 2 there are requirements wi for loading the parcels in time 1,2,…,14. Requirements in time 13 and 14 are not stated but they can be used for continuation of the task for time 15 and further. The requirement in time t is placed in the row wt. There is only one container available at each time with capacity W=10. In Table 1, there is a loading time listed for each parcel in rows BMa, FMa and SMa. The letter “L” in rows BMb, FMb and SMb means that a container is loaded in time corresponding the column (time). Number of used container are placed in column z.

The FMa and FMb rows contains loading times for parcels and containers gained from the FM model in the interval <1,7> and the SMa and SMb rows contain loading times for all parcel and containers gained from using the SM model in the interval <5,12>. For comparison, the standard BM model is used for the whole interval <1,12>.

The results show that FM model uses 2 containers (they are loaded in time 3 and 5) and SM model uses 4 containers (loaded in time 8,9,12 and 13). It makes 6 used containers during the whole interval. For comparison, the problem is also solved just with standard BM model on the whole interval <1,12> using the same number of used containers. But this solution contradicts with the assumption that information about parcels from the interval <8,12> is not known until the time 6, thus this solution does not suit the task.

For comparison of the results from FM and SM model (see Table 1), the problem is solved with the fundamental BM model on separate intervals <1,7> and <8,12>. The result is shown in the Table 2 which indicates that this solution (with 7 used containers) is worse than the solution in the Table 1, where both SM and FM models were used and the parcels from the end of the first interval are loaded in the second interval.

Table 1. Results of numerical experiments.

time t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	z
w_t	3	4	2	6	2	7	4	3	6	6	4	6	--	--	
FMa	3	3	3	5	5										
FMb			L		L										2
SMa						8	9	8	9	12	12	13			
SMb								L	L			L	L		4
BMa	3	3	3	5	5	8	9	8	9	11	13	13			
BMb			L		L			L	L		L		L		6

Table 2. Results of the based model for separate time intervals.

time t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	z
w_t	3	4	2	6	2	7	4	3	6	6	4	6	1	2	
BMa	3	4	3	4	6	6	7								
BMb			L	L		L	L								4
BMa								9	9	12	12	13			
BMb									L			L	L		3

5 Conclusion

This paper deals with container loading problem where parcels need to be loaded in the certain time interval. The aim is to obtain time schedule for loading parcels while minimizing the number of containers.

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