

# The Iterative Method of Development Pattern and Anti-Pattern

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**Abstract.** The ordering or ranking of multidimensional objects is a very popular subject of research. The first formal method was proposed by Hellwig [4]. He introduced the division of variables into stimulants (the bigger the better) and destimulants (the smaller the better). His main idea was to find the artificial object, called “development pattern” defined by the best values of variables. Then distances for all objects from the development pattern are calculated and objects are ranked according to the diminishing value of this distance. The next idea was to define the “development anti-pattern” – the artificial object defined by the worst values. Objects are ranked according to the growing distance from the anti-pattern. In the presence of outliers, at least in one variable, can have big influence on the final ranking. Such outliers introduce unexpected weighting system to the problem. If there is an object with one value very much bigger than the rest of the set, values of this variable – through standardization – are moved toward the other end of the distribution which becomes skewed. In order to avoid this problem we propose to use an iterative procedure. Objects are ranked one by one. We assigned just one rank to the best object on a given iteration of the procedure, and then this object is eliminated from the lot. The formula to calculate the composite index is given. The example deals with the measurement of innovation activity of provinces in Poland.

**Keywords:** Linear Ordering, Development Pattern, Multivariate Analysis.

## 1 Introduction

Linear ordering of objects based on composite indicator calculated from individual variables is used in different fields such as economy, quality of life, environment, development etc. The first method of linear ordering was proposed by Hellwig [4] in 1968. Generally not so many different approaches have been proposed in the literature since then. Saisana and Tarantola [11] list just six methods and they also give a review of some applications. TOPSIS approach (Hwang and Yoon [5]) and its fuzzy variants (Chen and Hwang [2]) can be added to this list. The crucial points in all procedures are as follows: selection of sub-indicators, normalization (or standardization) method, weighting system and aggregation formula. Uncertainty and sensitivity of choices with these steps has been studied by Saisana, Saltelli and Tarantola [10]. The most well-

known composite index is a Human Development Index for world countries, proposed by Amartya Sen and Mahbub ul Haq, calculated and published since 1990 (See Sen [12], Sager and Najam [9], Stanton [13])

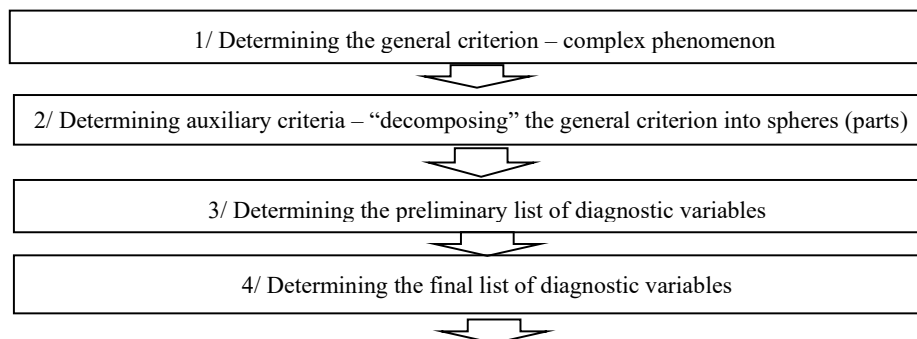
Markowska and Sokołowski [6] proposed an iterative method of multidimensional objects ordering using the most popular ordering method (OECD [8]), referred to in this paper as the classical one. The characteristics are made comparable by normalizing the distance from the worst object (the lowest value for stimulants and the highest for destimulants) by the range, for each characteristic separately. This operation brings down the values of all characteristics to the range [0;1] and changes destimulants into stimulants.

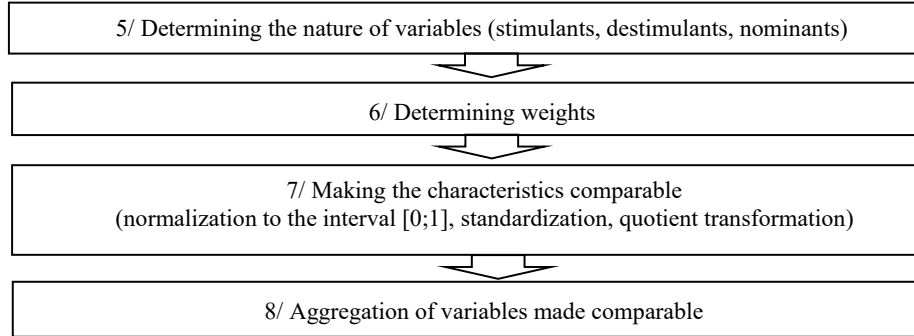
The aggregate index is calculated as an arithmetic mean, or a weighted arithmetic mean, if a reasonable weighting system can be proposed (e.g. expert weights). If the distributions of characteristics are very asymmetrical, or if outliers are present, an additional weighing factor is brought into the procedure. The importance of the negative asymmetric distribution is strengthened for the majority of characteristics, whereas those with strong positive asymmetric distribution – are weakened. The presented paper proposes defining the hierarchy of objects starting from the best one. Once assigned the subsequent rank, the object is removed from the working set, and thus the minimum or maximum reference points can be changed.

The goal of this study is to propose the application of an analogical iteration procedure to Hellwig's development pattern/anti-pattern method [4], and also to discuss the question regarding the extent to which the iterative procedure gives different results comparing to the original (non-iterative) Hellwig's approach. The empirical example provided has mainly an illustrative purpose, and not the comprehensive analysis of the problem considered.

## 2 Method

In the original development pattern method, firstly the characteristics are made comparable using standardization. Next the distances of objects to the development pattern defined by maximum stimulant values and minimum destimulant values, are calculated. The objects are ranked in a sequence from an object closest to the pattern to the one most distant from the pattern. The procedure of aggregate index calculation, in accordance with the classical ordering method, is presented in Fig. 1.





**Fig. 1.** Aggregate index determining procedure.

At the stage of step no. 7 the characteristics are usually unified, i.e. transformed into stimulants. The following formulas are applied in the classical method:

$$x_i^* = \frac{x_i - \min\{x_i\}}{\max\{x_i\} - \min\{x_i\}}, \quad \text{for stimulants, (1)}$$

$$x_i^* = \frac{\max\{x_i\} - x_i}{\max\{x_i\} - \min\{x_i\}}, \quad \text{for destimulants. (2)}$$

The next stage consists in the aggregation of variables made comparable – usually by using an additive formula taking the form of weighted arithmetic mean:

$$W_i = \frac{s}{\sum_{j=1}^m a_j} \sum_{j=1}^m a_j x_{ij}^*, \quad (3)$$

where:

$i$  – object's number,

$W_i$  – value of aggregate index for  $i$ -th object,

$j$  – characteristic's number,

$m$  – number of characteristics,

$a_j$  – weight of  $j$ -th characteristics,

$x_{ij}$  – value of  $X_j$  variable observed on  $i$ -th object (made comparable),

$s$  – scale factor (usually adopted as 1 or 100).

As a result, aggregate index values are calculated and can be ranked. Our proposal ranks objects individually, starting from the one “located” as the closest to the pattern. After finding the best object, it is eliminated from the list of the currently considered ones and standardization is carried out again. As a result, both the standardized values of characteristics and the pattern coordinates change. The following ranking positions are

determined gradually, one in each iteration, and later the assigned object is eliminated from the set in which the next consecutive object is searched for. This method can be referred to as the *moving pattern method*. The values of aggregate index are determined according to the procedure presented in Figure 2.

The described algorithm is repeated until three worst objects, assigned to the last ranks, remain in the subset. The procedure requires proposing a new method for determining the aggregate index, in which while determining the *i*-th rank the following is calculated:

$$D_{(i+1)} = \frac{W_{(i+1)}^i}{W_{(i)}^i}, \quad (4)$$

where:

$D_{(i+1)}$  – reduction ratio of aggregate index,

$(i)$  – *i*-th rank,

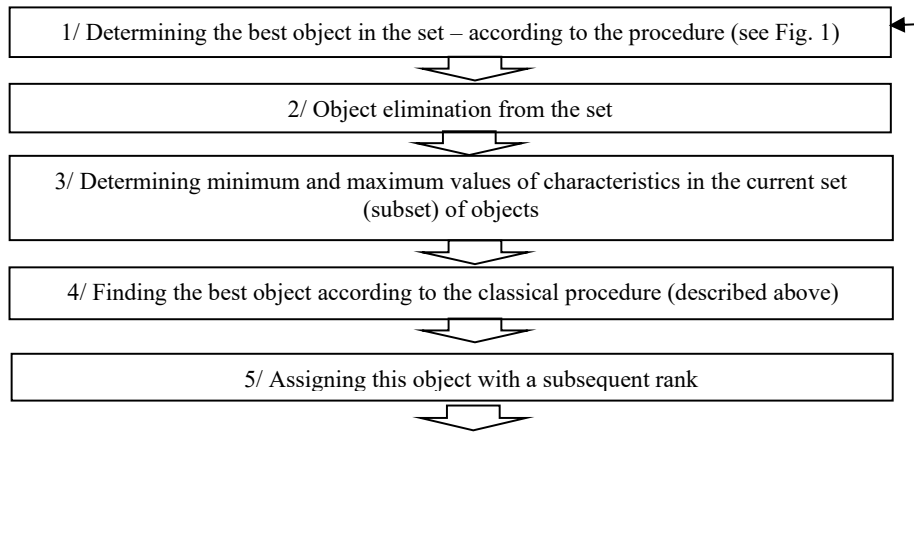
$W_{(i)}^i$  – local aggregate index calculated when determining *i* rank.

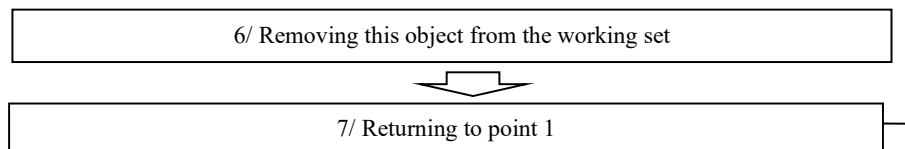
The final value of aggregate index is determined based on formula (5):

$$W_{(k)} = W_{(1)}^1 \prod_{i=2}^k D_{(i)}. \quad (5)$$

The values for subsequent objects are reduced according to the proportion resulting from the last joint comparison of the given and previous object (regarding the rank order).

The *moving anti-pattern method of development* works in a similar way, with the order of objects being determined from the worst object, i.e. the closest to development anti-pattern, determined by the smallest values of characteristics made comparable. Objects are removed from the set starting with *n* ranked object.





**Fig. 2.** Aggregate index determining procedure – *moving pattern method*.

Due to the fact that the purpose of the study is also to answer the question regarding the extent to which the iterative procedure gives different results than the original approach, one comparison is given here – based on statistical data used to develop the Relative Index of Enterprise Innovation Activity (RIEIA) proposed by Markowska and Strahl [7], in the part referring to industrial activity sphere (RIIEIA). The index covers 39 characteristics and a dual system of weights using expert weights and a hierarchical structure of characteristics. (Freudenberg [3] constructed an index of innovation performance based on 12 variables within three groups: generation of new knowledge, industry-science linkages / technology diffusion, industrial innovation).

The list of characteristics used in our example and the final values of weights are presented in Table 1.

**Table 1.** Variables used to develop the Relative Index of Industrial Enterprise Innovation Activity.

	Variable	Industry		Weight
		Range	Poland	
I	Enterprises investing in innovations	10.9-16.4	14.0	1.00
I	Average innovation investments per enterprise in 1000's PLN	356-1362	968	1.00
P	Enterprises receiving public financial support	3.6-7.5	4.9	1.00
P	Enterprises receiving public financial support from domestic institutions	1.6-4.9	2.7	0.50
P	Enterprises receiving public financial support from local authorities	0.4-2.1	1.0	0.25
P	Enterprises receiving public financial support from central authorities	1.1-3.0	1.9	0.25
P	Enterprises receiving public financial support from EU	2.5-5.9	3.7	0.50
P	Enterprises receiving public financial support from Horizon 2020	0.0-1.3	0.4	0.25
A	Innovation active enterprises	14.5-23.1	18.9	1.00
A	E.w.i.* innovations	13.7-21.5	17.6	1.00
A	E.w.i. new products	9.6-14.4	11.8	0.50
A	E.w.i. new processes	6.8-13.8	9.9	0.17
A	E.w.i. new logistics	1.8-4.7	3.2	0.17
A	E.w.i. supporting activities	4.0-7.5	5.9	0.17
A	E.w.i. organisational or marketing innovations	8.2-16.3	11.4	1.00
A	E.w.i. organisational innovations	5.3-10.5	8.1	0.50
A	E.w.i. new business practices for organisational procedures	3.6-8.2	6.1	0.17
A	E.w.i. new methods in work responsibilities	2.8-7.9	5.0	0.17
A	E.w.i. new methods in external relations	1.9-4.7	3.1	0.17
A	E.w.i. marketing innovations	4.7-12.5	7.1	0.50

Variable		Industry		Weight
		Range	Poland	
A	E.w.i. new packaging	2.6-9.0	4.2	0.13
A	E.w.i. new media and promotion	1.7-6.8	3.8	0.13
A	E.w.i. new product placement and sales	1.1-3.7	2.1	0.13
A	E.w.i. new pricing	1.4-5.4	2.7	0.13
C	Enterprises cooperating	3.7-8.4	5.5	1.00
C	Enterprises cooperating for receiving access to intellectual property	0.1-0.3	0.2	0.17
C	Enterprises benefiting from free intellectual property	0.7-2.7	1.4	0.17
C	Enterprises using innovations protected by exclusive rights	1.6-3.7	2.3	0.17
E	Revenues from products to the market	3.6-18.3	9.5	0.50
E	Revenues from products new to the firm	1.1-13.1	5.5	0.50
E	Enterprises with applications for trademarks in Poland	0.4-6.5	3.0	1.00
E	Enterprises with applications for industrial designs in Poland	0.5-5.3	1.3	1.00
E	Enterprises with applications for utility models in Poland	0.2-3.3	1.0	1.00
E	Enterprises with applications for patents in Poland	1.2-4.6	2.3	1.00
E	Enterprises planning to apply for foreign patents	0.1-2.8	0.6	0.50
E	Enterprises with Polish patent applications resulted from internal R&D activities	1.1-3.7	1.6	0.50
E	Enterprises which obtained patents in Poland	1.0-4.9	2.0	1.00
E	Enterprises which made application for foreign patent	0.3-3.4	1.0	1.00
E	Enterprises which obtained foreign patents	0.1-3.0	0.7	1.00

\* – *Enterprises which introduced*. Notes: I – investments, P – public aid, A – innovative activities, C – cooperation, E – effects; unless otherwise specified the values are expressed as percentage of all industrial enterprises [7].

### 3 Results

In the study [4] partial indexes were calculated for Polish regions at NUTS 2 level regarding such innovation areas as: investment outlays for innovation, public aid, innovative activity, cooperation in the field of innovation and the effects of innovative activities – for industry and separately for services. The statistical data used come from the short version of CIS (*Community Innovation Survey*) study [1], and are presented as percentages. The indexes were calculated using the classical normalization method to the interval [0;1], by a new iterative method, which defines the positions subsequently one after the other from the best object, and also using a rank (point-based) method. The cited study discusses such methodological problems as: the consequences of abandoning normalization (in the situation, in the case when the characteristics taken into account in the assessment are presented as percentages and represent stimulants), the selection of variable weighing systems (when they are of hierarchical nature and percentages do not add up to 100 due to the multiple choice of variants), aggregating partial indexes into a global one.

Below, along with the results of the original pattern and anti-pattern development methods and their iterative versions, the results of the classical linear ordering method and its iterative version are presented following Markowska and Strahl [7] – Table 2.

**Table 2.** The values of Relative Index of Industrial Enterprise Innovation Activity, calculated for Polish provinces, for 2015.

Province	Pattern method		Anti-pattern method		Classical method	
	original	iterative	original	iterative	original	iterative
Dolnośląskie	<b>25</b>	<b>23</b>	<b>63</b>	<b>66*</b>	<b>54</b>	<b>51</b>
Kujawsko-pomorskie	20	20	<u>45</u>	<u>33</u>	26	25
Lubelskie	15	14	<u>43</u>	<u>32</u>	<b>32</b>	<b>28</b>
Lubuskie	15	16	<u>41</u>	<u>31</u>	<b>26</b>	<b>18</b>
Łódzkie	<b>13</b>	<b>15</b>	<b>34</b>	<b>25</b>	24	24
Małopolskie	<b>32</b>	<b>30</b>	66	67	<b>49</b>	<b>46</b>
Mazowieckie	<b>27</b>	<b>25</b>	<b>56</b>	<b>50</b>	47	46
Opolskie	<b>26</b>	<b>24</b>	<b>69</b>	<b>73*</b>	50	49
Podkarpackie	<b>35</b>	<b>31</b>	<u>79</u>	<u>92*</u>	<b>62</b>	<b>57</b>
Podlaskie	57	57	115	115	64	64
Pomorskie	25	24	<b>54</b>	<b>48</b>	32	31
Śląskie	42	42	<b>75</b>	<b>78*</b>	59	59
Świętokrzyskie	<b>4</b>	<b>10*</b>	<b>25</b>	<b>18</b>	11	10
Warmińsko-mazurskie	17	17	<u>40</u>	<u>30</u>	29	28
Wielkopolskie	<b>27</b>	<b>25</b>	<u>54</u>	<u>44</u>	36	36
Zachodniopomorskie	<b>23</b>	<b>21</b>	<b>54</b>	<b>46</b>	<b>38</b>	<b>40*</b>

The change by two or more is marked in bold, the changes in plus are marked with an asterisks, and the changes by 10 or more are underlined. Source: author's calculations and [7].

When calculating an index value using the original pattern and anti-pattern development method, the selection of reference distance remains the key problem. Hellwig [4] proposed the reference point to be the mean distance plus twice the standard deviation of the distance. The reference point from the original version of the aggregate index does not ensure that the index value is less than 1 (and not higher than 100 in the version adopted for this study). Podlaskie province is close to the pattern, far from the anti-pattern and the index value for this province is relatively small in case of the pattern method, and relatively high in case of the anti-pattern method. The additional factor causing this phenomenon is the effect of "inflating space" – the larger the space, the longer the distances between empirical points. In case of the studied problem the classification space is a 39-dimensional one. Podlaskie province is the best one in industrial innovation activity, so by definition, its composite index has the same value in both, iterative and non-iterative procedures.

The similarity of index values calculated using classical methods against the results of iterative methods was assessed using linear correlation coefficients. They are presented in Table 3.

All of them are statistically significant (correlation coefficients from 0.826 up to 0.989).

**Table 3.** Linear correlation coefficients of innovation indexes.

Specification		Pattern method		Anti-pattern method		Classical method	
		original	iterative	original	iterative	original	iterative
Pattern method	original	1	<b>0.985</b>	0.967	0.931	0.884	0.901
	iterative	<b>0.985</b>	1	0.950	0.910	0.826	0.848
Anti-pattern method	original	0.967	0.950	1	<b>0.981</b>	0.908	0.910
	iterative	0.931	0.910	<b>0.981</b>	1	0.941	0.934
Classical method	original	0.884	0.826	0.908	0.941	1	<b>0.989</b>
	iterative	0.901	0.848	0.910	0.934	<b>0.989</b>	1

Correlation coefficients higher than 0.98 are marked in bold. Source: authors' calculations. While comparing the original versions with the iterative ones it can be observed that, in general, they are similar in case of three methods: pattern, anti-pattern and classical method (values in bold in Table 4). The relatively largest differences in the values of aggregate indexes occur in case of the anti-pattern method, and the results obtained using pattern method differ the most from the method referred to as the classical one. Table 4 lists the ranks of provinces based on the values of indexes from Table 2. The similarity of ordering was assessed using the Spearman's rank correlation coefficient (Table 5).

**Table 4.** Ranks of provinces based on the values of Relative Index of Industrial Enterprise Innovation Activity, calculated for Polish provinces, for 2015 [7].

Province	Pattern method		Anti-pattern method		Classical method	
	original	iterative	original	iterative	original	iterative
Dolnośląskie	<b>8</b>	<b>9</b>	6	6	4	4
Kujawsko-pomorskie	11	11	11	11	13	13
Lubelskie	<b>13</b>	<b>15</b>	12	12	<b>10</b>	<b>11</b>
Lubuskie	<b>14</b>	<b>13</b>	13	13	<b>14</b>	<b>15</b>
Łódzkie	<b>15</b>	<b>14</b>	15	15	<b>15</b>	<b>14</b>
Małopolskie	4	4	5	5	6	6
Mazowieckie	6	6	7	7	7	7
Opolskie	7	7	4	4	5	5
Podkarpackie	3	3	2	2	2	3
Podlaskie	1	1	1	1	1	1
Pomorskie	<b>9</b>	<b>8</b>	8	8	<b>11</b>	<b>10</b>
Śląskie	2	2	3	3	<b>3</b>	<b>2</b>
Świętokrzyskie	16	16	16	16	16	16
Warmińsko-mazurskie	12	12	14	14	12	12
Wielkopolskie	5	5	10	10	9	9
Zachodniopomorskie	10	10	9	9	8	8



**Table 5.** Rank correlation coefficients for provinces by innovation indexes.

Specification		Pattern method		Anti-pattern method		Classical method	
		original	iterative	original	iterative	original	iterative
Pattern method	classical	1	<b>0.988</b>	0.926	0.926	0.906	0.918
	iterative	<b>0.988</b>	1	0.909	0.909	0.859	0.879
Anti-pattern method	classical	0.926	0.909	1	<b>1.000</b>	0.956	0.956
	iterative	0.926	0.909	<b>1.000</b>	1	0.956	0.956
Classical method	classical	0.906	0.859	0.956	0.956	1	<b>0.991</b>
	iterative	0.918	0.879	0.959	0.959	<b>0.991</b>	1

Index values, according to the pattern and anti-pattern method can be combined into one indicator by an arithmetic or geometric mean. In the presented study it was decided to use the geometric mean. The results are presented in Table 6.

**Table 6.** The values of Relative Index of Industrial Enterprise Innovation Activity, calculated for Polish provinces, for 2015, obtained using combined pattern-anti-pattern method, and ranks of provinces.

Province	Index values		Ranks	
	original	iterative	original	iterative
Dolnośląskie	40	39	6	6
Kujawsko-pomorskie	30	25	11	11
Lubelskie	26	21	<b>13</b>	<b>14</b>
Lubuskie	24	22	<b>14</b>	<b>13</b>
Łódzkie	21	19	15	15
Małopolskie	46	45	4	4
Mazowieckie	39	35	7	7
Opolskie	42	42	5	5
Podkarpackie	53	54	3	3
Podlaskie	81	81	1	1
Pomorskie	37	34	<b>9</b>	<b>8</b>
Śląskie	56	57	2	2
Świętokrzyskie	10	14	16	16
Warmińsko-mazurskie	26	23	12	12
Wielkopolskie	38	33	<b>8</b>	<b>9</b>
Zachodniopomorskie	35	31	10	10

To sum up, it should be stated that, as in other applications of an iterative procedure, the ranking changes are quite small, but for local authorities even those small changes could be considered important, especially when the country consist of not so many administrative units of NUTS 2 level.

## 4 Conclusions

The example covering a relatively small number of objects was selected to illustrate the problem in order to allow the Reader, track changes in their ordering for each method.

In case of enterprises' innovation assessment against the classical version, the ranking changes were recorded for the following provinces:

- Dolnośląskie, Lubelskie, Lubuskie, Łódzkie and Pomorskie – in the iterative pattern method,
- Lubelskie, Lubuskie, Łódzkie as well as Pomorskie and Śląskie – in the iterative method.

All (applied) methods indicated that in 2015 the highest innovation activity was characteristic for industrial enterprises in Podlaskie province. It seems that the pattern-anti-pattern development method is a valuable one, both in the original and iterative version, as it combines the results of methods using two extreme reference points through the geometric mean.

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