

Modified Positional TOPSIS Method for Assessing the Socio-Economic Development Level of Rural Municipalities

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Abstract. Rural municipalities face many problems affecting their socio-economic development. These issues can be effectively addressed only if these units are assessed on a comprehensive basis. The socio-economic development is a complex phenomenon and its assessment requires taking into account many determinants. These features are often characterized by strong asymmetry or extreme observations. For the researchers, this is a considerable complication which cannot be dealt with using classic diagnostic methods because even a single outlier for a given object may significantly affect the analysis and the resulting conclusions. In view of the above, the relevant research should place a stronger focus on outlier-robust methods. Therefore, the objective of this paper was to use a modified positional TOPSIS method to assess the socio-economic development level of Polish rural municipalities in 2017. This approach reduces the adverse impacts of extreme skewness and outliers of features on the assessment of municipal development levels through the use of the spatial median of Weber and the extreme values identification method used in the Peaks over Threshold Model. Based on research, six types of Polish rural municipalities were identified in 2017. The socio-economic development level was found to vary strongly across municipalities. The highest levels were recorded in municipalities located near urban centers while the lowest levels were reported by remote municipalities.

Keywords: Modified Positional TOPSIS Method, Socio-Economic Development Level, Rural Municipalities.

1 Introduction

Territorial development is a long-term process of positive qualitative and quantitative changes. It includes economic, social and spatial processes affecting the territory considered. It is strictly related to local government units and may be examined at regional and local level. At regional level, development means an increase in the region's economic potential together with a permanent improvement in the region's competitiveness and in standards of living for the population [16]. Local development may be considered similar to regional development. The basic difference is that local

development takes place on a smaller scale and is restricted to a specific area: a small territory with its local residents. Hence, it involves changes occurring in a microregion, i.e. a local socio-territorial system which includes local administrative units (LAU).

In the US literature, local development is related to economic development [see e.g. 11, 14] while the European literature gives it a broader scope. Generally, local development means socio-economic development [see e.g. 15, 18]. Note that the assessment of socio-economic development relies on various quantitative and qualitative methods. On the one hand, analyses carried out using these methods allow for a better understanding of the socio-economic situation of local government units, which consequently can be helpful in planning their development. On the other hand, they are burdened with certain limitations resulting from assumptions and simplifications that can only partially explain the situation in a given local unit.

Parysek [13, p. 47] emphasizes that local development is a socio-economic development category which lacks strong tradition and is not backed up by well-established theories, methodologies or achievements, especially in Poland. "At least several dozen theories and concepts of various scopes, which emerged from different methodological approaches, provide a variety of explanations for the disparities in the pace of socio-economic processes across the territory" [12, p. 8]. Witkowski [21, p. 6] suggests that "each time, the direct reason for the establishment of a new approach were the disappointing results of implementing the previous one."

The socio-economic development level of administrative units is a complex aspect. It can be neither measured directly nor described with a single characteristic. To describe such aspects, synthetic measures are used which enable a summary description. The synthetic feature is based on many features which often are characterized by a strongly asymmetric distribution or extreme values. For the research process, this means a considerable complication which cannot be dealt with using classical methods for constructing a synthetic feature. This is because even a single outlier (very large or very small) for a given object may significantly affect the attribution of an excessively high (or low) rank in the final classification. In view of the above, the relevant research should place a stronger focus on outlier-robust methods.

The objective of this paper was to use a modified positional TOPSIS method to assess the socio-economic development level of Polish rural municipalities in 2017. The study population are rural municipalities which include only over rural areas. They were selected because they face many problems which can be effectively addressed only if these units are assessed on a comprehensive basis. This is the reason why the research on these areas is particularly important. The proposed research tool was the modified positional TOPSIS method which enables creating a synthetic feature in cases where the set of study variables includes features which are characterized by a strongly asymmetric distribution or extreme values. The study was based on 2017 data from the Local Data Bank of the Central Statistical Office [2].

2 Methodology

Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [6] is the most known technique for solving Multi-Criteria Decision Making (MCDM) problems. It is used to construction the ranking of objects describing by many features. This method has many advantages, particularly its classical version is simple to use. Classical TOPSIS and its modifications are popular approaches and have been widely used in various issues [1, 3, 9, 20].

We propose using the modified positional TOPSIS method to assess the socio-economic development level of rural municipalities. This method utilizes the mean excess function for identification of outliers (extreme values) of the features describing the level of socio-economic development of rural municipalities. The procedure to construct a synthetic measure, based on the modified positional TOPSIS method, includes six basic stages (Fig. 1).

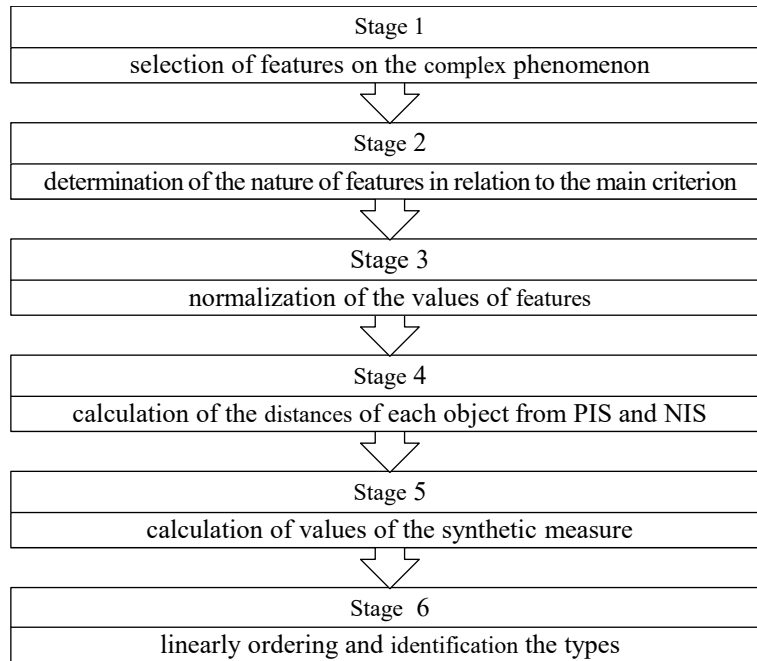


Fig. 1. Main stages of the procedure of constructing a synthetic measure.

The first stage contains the selection of features (indicators) describing objects (rural municipalities) and the second stage includes determination of the direction of their preferences in relation to the main criterion. The choice of features is based on substantive and statistical criteria. The set of features describing the level of socio-economic development of rural municipalities usually includes strongly asymmetric items or outliers. We propose to solve this problem by using:

- methods for identifying the threshold value in the Peaks over Threshold Model (POT) to establish threshold (limit) of outliers (extreme values),
- the positional formulation of TOPSIS method, that utilize the spatial median of Weber to limit the impact of the strong asymmetry of features.

First, methods for identifying the threshold value in the Peaks over Threshold Model [see e.g. 10] was used to establish limits of extreme values. In POT, the tail of the distribution of the feature is modeled using the Generalized Pareto Distribution (GPD). The beginning of the tail is determined by establishing a threshold value (ul_k). In this model, the starting point for the considerations is the conditional distribution of excess over ul_k of random variable X_k (k -th feature), defined as:

$$F_{ul_k}(y_k) = P(X_k - ul_k \leq y_k | X_k > ul_k) = \frac{F(y_k + ul_k) - F(ul_k)}{1 - F(ul_k)}, \quad (1)$$

where: $y_k = x_k - ul_k > 0$, F – the unknown distribution function of random variable X_k . According to the Pickands–Balkema–de Haan theorem, for a sufficiently large ul_k , the distribution function F_{ul_k} is definite and is well approximated by the GPD:

$$G_{\xi, \beta}(x_k - ul_k) = \begin{cases} 1 - (1 + \xi(x_k - ul_k)/\beta)^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp(-(x_k - ul_k)/\beta), & \xi = 0 \end{cases}, \quad (2)$$

where: $\beta > 0$, $x_k - ul_k \geq 0$ for $\xi \geq 0$ and $0 \leq x_k - ul_k \leq -\beta/\xi$ for $\xi < 0$. This distribution has two parameters: shape parameter (ξ) responsible for the thickness of the tail and scale parameter (β). Negative values of ξ show that the distribution has thinner tails than the normal distribution. In turn, the positive values ξ show that the distribution has fat tails, which is associated with an increased probability of extreme feature values. The correct choice of the threshold value ul_k is important, because it has an impact on the values of GPD parameter estimators. If N is the number of observations, N_{ul_k} is the number of observation in excess of ul_k , the estimator of the distribution function F is calculated from the formula:

$$\hat{F}(x_k) = 1 - \frac{N_{ul_k}}{N} \left(1 + \xi \frac{(x_k - ul_k)}{\hat{\beta}} \right)^{-1/\xi}. \quad (3)$$

The choice of the ul_k threshold should depend on the specifics of features and their number. The methods for selecting the threshold are presented by Coles [4] and in many other publications. One of the methods is to analyze the stability of estimated GPD parameters. The second method is to analyze the chart of the mean excess function. In this method, the starting point is the conditional expected value:

$$E(X_k - ul_k | X_k > ul_k) = \frac{\beta(ul_k)}{1 - \zeta}, \quad \zeta < 1. \quad (4)$$

Due to the fact that $\beta(ul_k)$ depends linearly on ul_k the empirical estimator of the conditional expected value also must depend linearly on ul_k . Hence the chart of the mean excess function:

$$\left\{ \left(ul_k, \frac{1}{N_{ul_k}} \sum_{i=1}^{N_{ul_k}} (x_{ik} - ul_k) \right) : ul_k < x_{k \max} \right\} \quad (5)$$

after exceeding ul_k should be linear. The lower limit (ll_k) of the feature is determined by performing calculations for the values of the feature multiplied by minus one. In this work, threshold values were selected on the basis of an analysis of chart of the mean excess function and the empirical density of features.

The selected features describing the level of socio-economic development of rural municipalities have a stimulating or destimulating effect on the socio-economic development (stage 2). Features that have stimulating effect are called stimulants, contribute to increasing the level of the socio-economic development. In turn, features that have destimulating effect are called destimulants, contribute to decreasing the level of the socio-economic development. Features that are destimulants may be transformed into stimulants with the use of a negative coefficient transformation.

Next, values of features are normalized using modified median standardization of Weber based on threshold values of features ul_k and ll_k ($k = 1, 2, \dots, K$) [cf. 7, 8]:

$$z_{ik} = \begin{cases} \frac{ll_k - m\tilde{d}_k}{1.4826 \cdot m\tilde{a}d_k} & \text{for } x_{ik} \leq ll_k \\ \frac{x_{ik} - m\tilde{d}_k}{1.4826 \cdot m\tilde{a}d_k} & \text{for } ll_k < x_{ik} < ul_k \\ \frac{ul_k - m\tilde{d}_k}{1.4826 \cdot m\tilde{a}d_k} & \text{for } x_{ik} \geq ul_k \end{cases}, \quad (6)$$

where: x_{ik} – value of the k -th feature in the i -th object ($i=1, 2, \dots, N$), $m\tilde{d}_k$ – Weber median (L1-median) vector component corresponding to the k -th feature, $m\tilde{a}d_k = med_i |x_{ik} - m\tilde{d}_k|$ – median absolute deviation of k -th feature values from the median component of the k -th feature, 1.4826 – a constant scaling factor corresponding to normally distributed data.

The median standardization of Weber is calculated for winsorized data. The median standardization of Weber should be used when the empirical distribution of the examined features is strongly asymmetric [7]. We adopt threshold values of features ul_k and ll_k ($k = 1, 2, \dots, K$) in winsorization of data. Winsorization is a process of

replacing a determined number of extreme values of features with a constant (smaller or bigger) value.

In fourth stage the positive ideal solution (PIS) [see e.g. 23]:

$$A^+ = \left(\max_i(z_{i1}), \max_i(z_{i2}), \dots, \max_i(z_{iK}) \right) = (z_1^+, z_2^+, \dots, z_K^+), \quad (7)$$

and negative ideal solution (NIS) were calculated:

$$A^- = \left(\min_i(z_{i1}), \min_i(z_{i2}), \dots, \min_i(z_{iK}) \right) = (z_1^-, z_2^-, \dots, z_K^-) \quad (8)$$

PIS includes the maximum (ideal – the best) values of each feature. Whereas, NIS are the minimum (anti-ideal – the worst) values of features.

In next step distances for each object from PIS (A^+) and NIS (A^-) were calculated based on the median absolute deviation (stage 4) [23]:

$$d_i^+ = \text{med}_k \left(|z_{ik} - z_k^+| \right), \quad d_i^- = \text{med}_k \left(|z_{ik} - z_k^-| \right), \quad (9)$$

where med_k – marginal median for the k -th feature.

In fifth stage a synthetic measure is constructed based on Hwang's and Yoon's idea [6]:

$$S_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad (i = 1, \dots, N), \quad 0 \leq S_i \leq 1. \quad (10)$$

The closer to 1 is the value of the synthetic measure, the higher level of development. Hellwig's method [5], Strahl's method [19] and non-model methods based on sum, mean or weighted mean are examples of alternatives to TOPSIS.

Established values of the synthetic measure are used in rank ordering of rural municipalities. Next, they allowed to identify typological classes of rural municipalities (stage 6). Identification of classes for the entire range of variation of a synthetic measure may be performed using statistical methods or arbitrary manner. In the study, it was applied an arbitrary approach based on numerical ranges of values for synthetic measure (Tab. 1).

Table 1. Identification of classes by arbitrary manner.

Class	1	2	3	4	5	6
Level of development of the phenomenon	very high	high	medium-high	medium-low	low	very low
S_i	(0.80; 1.00)	(0.60; 0.80)	(0.50; 0.60)	(0.40; 0.50)	(0.20; 0.40)	(0.00; 0.20)

3 Results of Research

The first step in diagnosing the state of socio-economic development at local level is to identify a set of features which characterize spatial structure in the cross-section of rural municipalities. The features were selected based on substantive and statistical analysis. Having analyzed the socio-economic situation of the rural municipalities, the set of 17 features was selected for Polish rural municipalities ($N=1555$) to describe them in terms of socio-economic development. The features were grouped by five criteria: demographic and social situation, technical infrastructure, social infrastructure, economy and public finance. The criteria are represented by the following features:

Criterion 1. Demographic and social situation: share of the registered unemployed persons in the population in the working age (%) (x_1), non-working age population per 100 persons of working age (x_2), old-age dependency ratio – the number of persons aged (65 and over) per 100 persons of working age 15–64 (x_3),

Criterion 2. Technical infrastructure: water supply network per 100km² (km) (x_4), sewage network per 100km² (km) (x_5), share of persons using wastewater treatment plants in total population (%) (x_6), dwelling stock per 1000 population (x_7),

Criterion 3. Social infrastructure: bed places per 1000 persons (x_8), pupils per section in primary schools (x_9), population per generally available pharmacy (x_{10}),

Criterion 4. Economy: entities entered in the REGON register per 10 thous. population (x_{11}), share of agricultural holdings with an area 15 ha and more in total number of agricultural holdings (%) (x_{12}), share of agricultural land in total lands (%) (x_{13}), share of built-up and urbanized areas in total area (%) (x_{14}),

Criterion 5. Public finance: financial self-sufficiency of municipality – share of own incomes in total incomes (%) (x_{15}), own incomes per capita (PLN) (x_{16}), share of investment expenditure in total expenditure (%) (x_{17}).

The studies are based on statistical data from 2017 coming from the Local Data Bank of the Central Statistical Office of Poland [2].

Five-year (2013–2017) means were calculated for feature x_{17} representing the share of investment expenditure in total expenditure. In the analysis, it was assumed that three features (x_1 , x_2 and x_3) are destimulants, two features (x_9 and x_{10}) are nominants (nominant is the type of feature which is stimulant in some range of a feature and destimulant in other its range) while other are stimulants.

Table 2 presents descriptive statistics of the features – indicators of rural municipalities in Poland in 2017 and the threshold values for the features, calculated based on a graph analysis. The features demonstrated positive skewness, except x_{13} which had negative skewness. Extremely high skewness was observed for x_2 , x_3 , x_5 , x_8 , x_{11} , x_{14} and x_{16} . The distributions of ten features (x_2 , x_3 , x_4 , x_5 , x_7 , x_8 , x_9 , x_{11} , x_{14} , x_{16}) had high positive kurtosis which means a high probability of extreme values. The distributions of these features had fat right tails. In turn, for each feature, the left tail of the distribution was thinner than that of a normal distribution or was cut off.

In order to limit the influence of outliers, the limits of outliers of features were established (Tab. 2). The calculations were performed with *fExtremes* in R [22]. Afterwards, winsorization was performed to replace tail values with calculated threshold values. Once transformed, the distributions of the features exhibited at most

moderate skewness. Only in the case of x_8 , the distribution continued to have extremely high positive skewness, though considerably reduced. Next, features with a destimulating effect were converted into stimulats.

Table 2. Descriptive statistics and threshold values of features of rural municipalities in Poland in 2017.

x_k					St.			Ex.		
	Mean	Med	Max	Min	dev.	Mad	Sk.	kurtosis	ll_k	ul_k
x_1	5.5	4.9	18.2	1.0	2.8	1.7	1.1	1.5	×*	12.0
x_2	60.7	60.3	113.4	45.7	4.9	2.8	2.0	16.6	51.4	70.3
x_3	22.6	21.9	71.8	11.7	5.0	2.9	2.2	14.8	14.1	30.3
x_4	100.4	90.5	533.4	0.0	60.9	33.6	1.6	5.4	×	185.7
x_5	45.4	24.7	656.7	0.0	64.7	18.8	3.4	15.5	×	111.4
x_6	40.1	35.4	100.0	0.0	28.1	20.1	0.4	-0.8	×	×
x_7	311.9	304.2	665.6	203.2	50.4	27.5	1.8	7.2	235.7	403.1
x_8	10.7	0.0	1123.3	0.0	44.7	0.0	14.5	295.0	×	45.8
x_9	15.1	15.1	54.1	6.8	3.0	2.0	1.4	17.1	8.8	21.3
x_{10}	3425.2	3332.0	18931.0	0.0	3015.4	2162.0	1.0	1.7	×	10527.0
x_{11}	710.9	659.1	3597.8	277.8	262.8	126.0	3.1	19.9	384.1	1107.1
x_{12}	12.0	9.0	51.8	0.0	10.9	7.2	1.0	0.3	×	41.6
x_{13}	66.0	69.3	96.5	5.5	17.9	11.4	-0.8	0.1	24.1	95.9
x_{14}	4.0	3.3	35.7	0.7	2.8	0.7	4.4	28.7	1.5	6.4
x_{15}	33.6	30.9	92.7	12.4	12.3	7.3	1.0	1.2	×	60.6
x_{16}	1550.4	1318.8	43781.8	537.5	1459.1	311.0	18.8	495.5	×	2867.3
x_{17}	15.0	14.4	56.1	2.4	5.9	3.7	0.9	2.1	×	29.4

*Not determined threshold.

In third stage, the values of indicators were standardized using the modified median standardization of Weber. The calculations were performed with *robustX* in R [17]. The standardized values of features allowed us to calculate the distance of each rural municipality (LAU2) considered from the PIS and the NIS with the use of the median absolute deviation. Next, the values of the synthetic measure of socio-economic development of rural municipalities were calculated using the modified positional TOPSIS method. The range of variation of the synthetic measure is from 0.096 to 0.957, which allowed to more accurately determine the rank and types of rural municipalities (including six levels of socio-economic development of rural municipalities, from “very high” to “very low”) (Tab. 3).

Table 3. Typological classification of rural municipalities in Poland in terms of the level of socio-economic development in 2017.

Class (c)	Level of socio-economic development	S_i	N_c	%
1	very high	(0.80; 1.00)	22	1.41
2	high	(0.60; 0.80)	115	7.40
3	medium-high	(0.50; 0.60)	232	14.92
4	medium-low	(0.40; 0.50)	484	31.13
5	low	(0.20; 0.40)	665	42.77
6	very low	(0.00; 0.20)	37	2.38

N_c – the number of objects in c -th class.

As shown by the empirical study, typological class with very high level of socio-economic development collected the 1.41% of the total number of rural municipalities in Poland. Class first, demonstrating a very high level of socio-economic development, was composed of twenty two municipalities. Most of them are located in the immediate vicinity of the cities. The proximity of the urban centers provides the rural municipalities with socio-economic benefits. The suburbanization process results in a dynamic development of the residential function of rural municipalities located in the first ring around large urban centers. The consequence is the development of infrastructure. In 2017 the first class was characterized by the highest share of own incomes in total incomes (more than 60%), the highest level of own incomes per capita (PLN 2867.3), the highest share of investment expenditure in total expenditure (24.2%). In these municipalities, the number of entities entered to the REGON register per 10 thous. population was nearly twice the Polish mean (over 1107). Also share of agricultural holdings with an area 15 ha and more in number of total agricultural holdings had the highest level (11.3%) (Tab. 4).

Table 4. Intra-class mean* values of features of socio-economic development levels of rural municipalities of Poland in 2017.

Feature	Class						Poland**
	1	2	3	4	5	6	
x_1	2.4	3.2	3.9	4.8	5.7	7.2	4.9
x_2	59.4	58.5	58.2	60.1	61.5	65.6	60.3
x_3	15.4	18.9	20.3	21.6	23.9	28.3	21.9
x_4	181.4	131	105.1	94.5	79.5	72.8	90.5
x_5	111.4	88.8	45.8	28.9	12.3	1.5	24.7
x_6	88.6	68.8	52	43.1	23.4	6.6	35.4
x_7	351.3	309.6	303	300.5	304.9	304.5	304.2
x_8	12.2	6.1	3.3	0.0	0.0	0.0	0.0
x_9	19.4	16.6	15.3	15.0	14.8	12.8	15.1
x_{10}	3229.3	4062	3417.6	3740	2948	0.0	3332
x_{11}	1107.1	1013.4	817.0	695.5	573.5	462.4	659.1
x_{12}	11.3	5.6	9.2	9.7	9.6	5.9	9.0
x_{13}	67.2	69.3	67.4	68	70.9	74.2	69.3
x_{14}	6.4	6.4	4.4	3.5	2.8	2.4	3.3
x_{15}	60.6	52.5	43.2	33.1	25.7	18.7	30.9
x_{16}	2867.3	2390.8	1835.5	1401.9	1085.1	797.5	1318.8
x_{17}	24.2	18.5	16	14.6	13	11.4	14.4

* The mean values of features are represented by their medians. ** The medians were calculated for values of features for all rural municipalities of Poland.

The second type shows high socio-economic development of 115 rural municipalities. In 2017 the class of rural municipalities was well developed in technical infrastructure and had very good demographic and social situation. In these municipalities, a large number of registered entities entered to the REGON register per 10 thous. population was also observed (over 1013, the mean for rural municipalities in Poland being ca. 659) (Tab. 4).

Average levels (medium-high and medium-low) of socio-economic development were reported by 716 rural municipalities grouped in the third and fourth typological classes (14.92% and 31.13%, respectively). In these municipalities, the majority of indicators of socio-economic development had similar to average values of features in rural municipalities in Poland. Note also that the technical infrastructure (water supply system, sewage system, wastewater treatment plants) was developed better than the average in rural municipalities in Poland (Tab. 4).

In turn, the fifth typological class, demonstrating low level of socio-economic development, was composed of 665 (42.77%) rural municipalities of Poland. These municipalities are at lower levels of technical and social infrastructure development. Rural municipalities of the fifth class reported a distinctively low level of financial self-sufficiency (less than 26%) (Tab. 4).

The sixth class with very low level of socio-economic development collected barely 2.38% of the total number of rural municipalities in Poland. These municipalities exhibited pronounced characteristics of remote areas, reflected by economic stagnation. It was noticed relatively small number of entities entered to the REGON register per 10 thous. population (ca. 462). These rural municipalities were characterized by the lowest level of own incomes per capita (barely PLN 797.5) and, at the same time, the lowest share of own incomes in total incomes (less than 19%). In these municipalities, the low levels of own incomes per capita and low shares of own incomes in their budgets resulted in the lowest share of investment expenditure in the total expenditure. This contributed to very low socio-economic development level of the rural municipalities. Also, these municipalities had a poorly developed social and technical infrastructure which exhibited certain deficiencies. In these areas, no year-round accommodation was offered and no pharmacies were available. These were mainly agricultural municipalities located away from large urban centers. The dominant role of agriculture was manifested by a considerable share of agricultural land in total land area (over 74.2%). Note however that agricultural land was mainly distributed between small farms as the share of 15 ha or larger farms did not exceed 6% (Tab. 4).

4 Conclusions

If the set of feature values includes outliers, constructing the synthetic measure with the use of classical methods may result in excessively reducing the range of variation of the synthetic development indicator. As a consequence, it may become problematic to properly identify the development types of the complex aspect under consideration. The reason for these problems is that empty classes may appear if classical methods for identifying development types are used. Moreover, in the final ranking, an excessively high or low rank may be attributed to objects whose observed features include outliers. To solve these issues, an outlier-robust approach was proposed. As shown by empirical studies, the proposed approach based on the modified positional TOPSIS method properly reflected the differences in socio-economic development levels between classes of units covered by the study.

The modified positional TOPSIS was used to synthetically assess and identify the types of socio-economic development levels of rural municipalities in Poland. This is a suitable approach to determine the synthetic development measure in a case where the set of features of municipalities includes outliers or strongly asymmetrical values. The modified positional TOPSIS method is robust to outliers and to defined values of the positive ideal solution and the negative ideal solution.

The study enabled the identification of six types of socio-economic development level (from very high to very low). The largest class comprises rural municipalities at low levels. In turn, classes at very high and very low development levels are poorly represented among rural municipalities. Note also that the most developed municipalities are located near cities whereas remote areas are the least developed ones. In other words, the greater the distance from urban centers, the lower the development level of rural municipalities.

The proposed approach to assess the socio-economic development levels of rural municipalities in Poland is a universal approach that may be used for other administrative units. The proposed research approach may also be the basis for the establishment of development documents, e.g. development strategies or programs of development.

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